

The Effect of Problem-Solving Approach on Undergraduate Students' Achievement in Mathematical Thinking

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Abstract: This study investigates the effects of the problem-solving approach on students' achievement in mathematical thinking. Fifty undergraduates from the mathematics program in a public university in Klang Valley participated in a quasi-experimental design study. 25 participants formed the experimental group, who experienced a problem-solving approach for eight weeks of intervention, and the other 25 (control group) were taught in a traditional manner. The data were collected over time using two achievement tests (pre-intervention and post-intervention). A split-plot analysis of variance (SPANOVA) was conducted to assess the effect of both groups on mathematical thinking achievement across the two time periods. The results indicated that the intervention was found to improve the achievement of the experimental group in the mathematical thinking test. Comparison the post-test results showed that participants who experienced the intervention achieved better results than the control group, with $p < 0.05$. The principles and practical applications of fundamental mathematical thinking processes should be taught in accessible ways, especially to high school graduates or novice undergraduates, to ensure they can apply mathematical thinking in the future.

Keywords: Achievement, Heuristics, Mathematical Thinking, Problem-solving Approach, Metacognitive

1. Introduction

The essence of mathematical thinking is inextricably linked to the cognitive processes which generate mathematical knowledge. One of the ways to ensure that students are involved with mathematical thinking is through non-routine problems. Mathematical thinking requires non-routine or unfamiliar mathematical problems so that students can flexibly include their understanding of mathematics' fundamental concepts and ideas, and focus on the problem-solving process (English & Kirshner, 2016; Schoenfeld, 1992). Non-routine problems are the kinds of problems that contribute to students' mathematical problem solving and reasoning skills. According to Hershkowitz et al. (2001), if the students solve a routine problem, they will alternate between recognising and building with previously acquired structures. If they solve a non-routine problem, they can develop and reflect on a new (for them) phenomenon, its internal structure, and its external connection to items they already know. Even though routine problems can serve essential didactic functions of teaching students to apply

a certain method or a definition in mathematics correctly, they can only improve their problem-solving skills by using non-routine problems (Stanic and Kilpatrick, 1988).

The effect of a problem-solving approach on undergraduate students' achievement in mathematical thinking has been extensively studied, revealing significant benefits in both performance and attitude towards mathematics. This approach focuses on problem-solving tasks, which enhance students' understanding and application of mathematical concepts. Recent studies have indicated that students taught using a problem-solving approach outperform those taught in a traditional method. Albay (2020) found the first-year college students improved tremendously in the College Algebra performance when engaged in problem-solving tasks. Mathematical thinking also can be enhanced with the problem-solving heuristics (Singh et al. (2018). The problem-solving approach also encourages critical thinking and higher-order thinking skills among students as highlighted in Hasan (2024) and Tambunan (2019). While the studies above have shown positive outcomes, some educators argue that it may not address all learning styles effectively, suggesting a need for a balanced pedagogical strategy to cater to the diversified students' needs.

The mathematical problem-solving skills of Malaysian students remain unsatisfactory. Even at the university and college levels, many students struggle to recognize when their final answers or solutions are logically incorrect. According to Singh et al. (2024), a significant cognitive gap in mathematical thinking, particularly in problem-solving, exists among high school graduates entering college. This issue primarily stems from students' inability to apply previously learned mathematical knowledge when solving problems. Hoon et al. (2018) further support this finding, noting that most university students rely solely on procedural knowledge without engaging in reasoning when solving mathematical problems. One of the key factors contributing to this challenge is the way mathematics is taught in schools—often as a rigid, procedure-focused subject rather than one that fosters critical thinking and application (Nasir et al., 2021). Consequently, students struggle to develop essential problem-solving skills, particularly in transforming information based on relationships, which has been identified as a major obstacle in mathematical problem-solving (Hoon et al., 2020). As a result, many college students gradually adopt rote learning strategies, and their final grades do not accurately reflect their development in mathematical thinking or their ability to solve problems effectively (Singh et al., 2016).

Other than that, the students could not describe their argument in a meaningful way other than specifying the algorithmic procedure processes. This phenomenon indicates that they are ignorant of their thinking when solving the problem. Singh (2017) has found that students are given the solution based on the direct assumption rather than proven through mathematical and logical reasoning. These difficulties and mistakes are encountered because many undergraduate students are too focused almost entirely on formal mathematical algorithms, principles and procedures that appear to be highly distant from conceptual comprehension (Singh et al., 2018; Bowyer & Darlington, 2016). Thus, the students failed to comprehend the fundamental of formulaic structures in their mathematical learning. Han et al. (2016) have found that some students can understand the theoretical concepts involved in the problem. However, they do not know how to solve the problem. The students are found too dependent on formula and procedural knowledge and make them struggled to integrate with the suitable strategy or prior knowledge when solving the uncommon problem (Nasir et al., 2021; Han et al., 2016; Singh & Hoon, 2017; Hoon et al., 2018). Besides, the students were found to lack alternative strategies even though they struggled to remember the mathematics formula to solve the problem. Students did not show a second attempt or further effort to develop a solution (Hoon et al., 2018) and felt it was difficult to apply their mathematical knowledge in unfamiliar contexts (Nasir et al. 2021; Bowyer & Darlington, 2016).

In addition, current research has found that students are not aware of their thinking process when solving the problem. According to Singh and Hoon (2017), the students could easily apply computation and algorithm. However, they just used the method by neglecting logical thinking and did not realise the answer given did not make sense. Furthermore, the students rarely looked back to assess if the path they had chosen was productive and, if not, to consider other options. In other words, the students did not monitor and, when necessary, control their thinking processes in the way that successful problem solvers do (Wedelin et al., 2015). This finding is supported by (Zakaria, Yazid, & Ahmad, 2009) and Abdullah et al. (2017), who found that the level of self-checking among school leavers and school students when solving the problem is still at a moderate level.

Due to these reasons and facts that have been discussed above, it is important to develop students' thinking processes. Generally, research indicate that knowledge and awareness of one's thinking strategies establish relationships with the ability of solving problems (Schoenfeld, 1992; Schoenfeld, 1994; Safari & Meskini, 2016). These thinking strategies relate to students' cognitive self-regulation or monitoring skills when solving the problem. This term "monitoring and control" refers to the aspect of metacognition known as self-regulation. Schoenfeld (1992) described self-regulation or "monitoring and control" when explaining problem-solving and mathematical thinking. Therefore, concerning the importance of thinking processes, the first component emphasised in this program was cognitive-metacognitive strategies.

Various researchers have suggested a need for a program or initiative to boost students' mathematical thinking, especially at the tertiary level (Hoon et al., 2018; Singh et al., 2016; Schoenfeld, 1992). According to Viitala (2017), the students need to be aware of their learning and problem-solving processes when learning mathematics. It could help them to cope in new situations and develop their metacognitive skills not only in mathematics but also in other subjects. Besides, the teaching methodology used by the mathematics instructors in mathematics classes also relates to developing students' thinking processes (Mevarech & Fan, 2018). Therefore, in this study, we aimed to use the problem-solving approach to enhance students' cognitive-metacognitive strategies towards the development of their mathematical thinking. Our target is to apply problem-solving approach to elicit students' thinking processes where the students are exposed and engaged in various tasks and problems. The emphasis was on exploring different mathematics contexts through the application of problem-solving stages (Polya, 1973 and Schoenfeld, 1992), problem-solving strategies and cognitive-metacognitive strategies while solving the task given. Thus, the prime aim of the study was to investigate the effect of the problem-solving approach on students' development of mathematical thinking with the objectives:

- a) To determine undergraduate students' initial (current) level in mathematical thinking test before the intervention of problem-solving approach.
- b) To determine whether the problem-solving approach significantly increases learners' achievement in mathematical thinking.

2. Conceptual Framework

Polya (1973) defines problem-solving as the process used to solve a problem without an obvious solution. It is a cognitive process aimed at achieving a goal for which the pupils do not have a primary solution strategy (Rahman, 2019). Polya (1973) has suggested a four-phase problem-solving model: 1) understanding the problem; 2) devising a plan; 3) carry out the plan; 4) look back. Subsequently, Schoenfeld (1992) refined Polya's problem-solving principles by introducing six problem-solving phases, which are read, analyse, explore, plan, implement, and verify. Identification of a problem statement and forming knowledge contextually are the first steps towards solving issues. It emphasises the importance of students recognising the specific problem(s) to be solved, designing and implementing a solution, and monitoring and evaluating progress throughout the process. The problem involved is unlike a straightforward task. It implies a non-routine state or circumstance for which there are no readily available standard solutions (Mayer & Wittrock, 1996). As a result, the problem necessitates logical reasoning to process detailed information that may be used to successfully and efficiently address the problem.

Schoenfeld (1992) has suggested that problem-solving strategies and metacognition are related to students' cognition in mathematics. Thus, problem-solving strategies are also known as heuristics. Heuristic aims to study the methods and rules of discovery and invention in solving the problem (Polya, 1973). According to Watters and Logan (2006), gradually guiding students with different strategies could prepare them to be more competent and confident in solving the problem. Current studies have found that proficiency and knowledge in strategies relate to students' success in solving problems (Nasir et al., 2021; Singh et al., 2016; Singh et al., 2018). Metacognition is a part of problem-solving that incorporates pupils' knowledge of what they're thinking or what's going on in their heads as they solve the problem (Downing et al., 2009). Metacognition involves an individual's awareness of his cognitive strategies, which refers to students' planning and monitoring their problem-solving process and

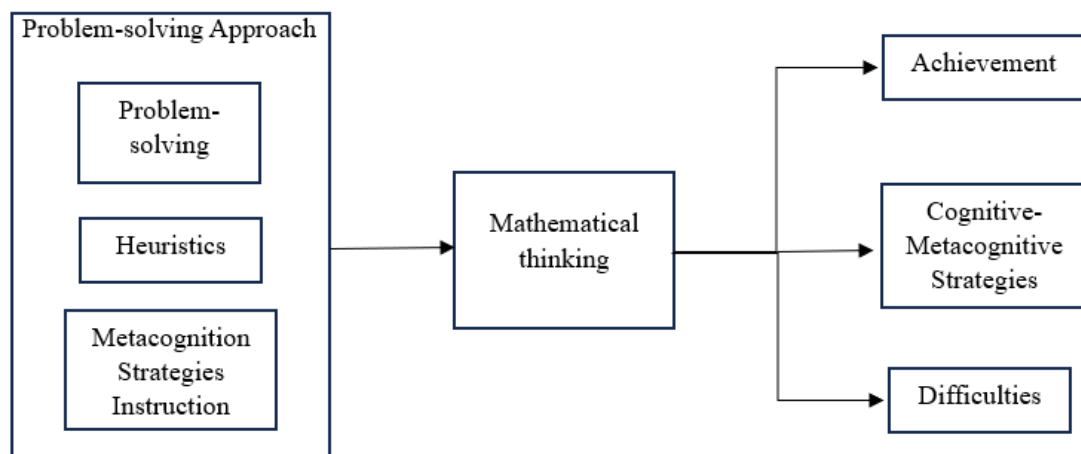
consciousness about their strategies used to solve the problem (Zakaria, 2003). Past research have indicated that low proficiency in metacognition leads to poor problem solver (Smith & Mancy, 2018; Abdullah et al., 2017; Schoenfeld, 1992).

Metacognitive and problem-solving strategies development can ensue through explicit teaching or training (Mevarech & Fan, 2018). Mevarech and Fan (2018) have found that metacognition and problem-solving strategies are teachable. They suggested that significant components of learning metacognitive and problem-solving strategies are through explicit teaching and intense practice of metacognitive processes. Improving students metacognitive and problem-solving strategies are essential, as they will develop students ability to solve various problems successfully. Problem-solving, which emphasises the usage of metacognition and applying different techniques, plays a vital role for students to be engaged in mathematical thinking (Drijvers et al., 2019; Mason et al., 2010).

Thus, this study considered all of the above and concluded with the framework in Figure 1.

Figure 1

Conceptual Framework



3. Methodology

3.1 Research Design

The present study employed a quasi-experimental design involving a sample of 50 undergraduates enrolled in a first-year in mathematics program in a public university in Klang Valley. Two intact groups from the same university and program were selected as the experimental and control. The experimental group experienced a problem-solving approach (PSA) intervention for eight weeks, while the control group only received a regular classroom discussion as a comparison group. First, the pre-test was conducted before the intervention process. Then, the post-test was done a week after the end of the intervention. Then, the test scores and the ability of these groups (experimental and control) in pre-test and post-test were compared to determine the effect of the intervention.

3.2 Participants

The participants involve 50 first-year undergraduate students in a mathematics program in a public university in Klang Valley. The researchers used simple random sampling to select two intact groups among first-year undergraduates in a public university in Klang Valley to become experimental and control groups. The selection of these two intact groups adheres to university rules and policies. Furthermore, randomly assigning students to different groups is challenging and disrupts the original classroom setting and learning experiences. Therefore, the participants are only selected if they consent

and show willingness to participate in the study. The experimental and control groups are given similar tasks, questions and answer schemes. However, only the experimental group received eight weeks of problem-solving approach intervention. The details of participants' backgrounds are represented in Table 1.

Table 1

Demographic Data of Participants

Type							
Gender	Experimental	Control	Total				
Male	4	4	8				
Female	21	21	42				
Total	25	25	50				
Grade (Modern Mathematics)				Grade (Additional Mathematics)			
Grade	Experimental	Control	Total	Grade	Experimental	Control	Total
A	25	25	50	A	5	4	9
				B	9	10	19
				C	11	11	22
Total	25	25	50	Total	25	25	50

3.3 Instrumentation

Participants' mathematical thinking achievement was measured through the Mathematical Thinking Test (MTT) constructed by the researchers. Two types (pre and post) of MTT were constructed to measure the participants' mathematical thinking before and after the intervention. The MTT consists of 10 non-routine problem items that involve a variety of fundamental mathematics topics and areas from primary to secondary levels such as Numbers & Arithmetic, Number theory & Combinatorics, Geometry, Algebra and Logic. The teaching and learning of mathematics courses is conducted in English in most universities and colleges under the Ministry of Higher Education and as such, the MTT was conducted in English.

In determining its content validity, the constructed MTT items (pre and post) were submitted to seven experts to be evaluated. All the experts are experienced and have background in mathematics education, problem-solving and mathematical thinking. In addition, the experts rated certain aspects of the items, including language suitability, understandable sentences, clear item meaning, questions that are suitable or parallel to the topics, and applicable of the question involved with thinking. After the evaluation, the test items were edited and reviewed once again. The focus of the MTT was placed on participants' overall performance and their ability in heuristics knowledge when solving the tests. A marking scheme of the MTT consists of the solution and individual score items prepared and evaluated together by the experts. The tests (pre and post) were tested with a reliability test with a high-reliability index, r , 0.859 and 0.894 for pre-test and post-test, respectively.

3.4 Intervention

The experimental group received interventions of a problem-solving approach (PSA). The intervention aimed to help prospective undergraduate students enhance their thinking process in solving mathematics problems. Through the intervention PSA, these students were explicitly taught problem-solving as a mathematical concept. More specifically, the intervention of PSA incorporates: (a) problem-solving stages [as suggested by Polya (1973) and Schoenfeld (1992), problem-solving strategies [as suggested by Tjong et al. (2005)], and metacognitive strategies (instructor) [as suggested by Ozsoy & Ataman (2013)]. The development of the intervention was guided by several considerations gained from various literature. These considerations include:

- a) Problem-solving stages: the primary purpose of problem-solving is not to equip students with a collection of skills and processes but rather to think for themselves. Solving a problem is not a linear process.
- b) Metacognition: Students responsible for monitoring their own thoughts. We believed that metacognitive behaviour could be taught systematically through problem-solving activities. Such learning
- c) Problem-solving strategies: There are various strategies used in solving mathematics problems.
- d) Metacognition as part of mathematics instruction: The instructor should involve active learning on the role of students learning. The activities and strategies could create a conscious reflection upon students' thoughts and ideas as the essence of metacognition.

Due to these considerations, the intervention has implemented explicit instruction that uses metacognitive strategies as the instructional strategies to guide and prompt the participants to solve the problem. PSA emphasised the role of cognitive-metacognitive strategies by implementing the six problem-solving stages (read, analyse, exploration, plan, implementation, verification) and heuristics when solving problems. This program encourages students to participate in various exercises, problems, and investigations as they explore mathematics concepts from a problem-solving perspective in an interactive manner. PSA emphasised the exploration of different mathematics contexts to learn mathematics, pose problems and problem extensions, solve problems, and communicate mathematical demonstrations

4. Findings

Before discussing the results, both tests (pre-test and post-test) were examined for their normality tests through Shapiro-Wilk (since the number of samples in each group is less than 30). Each test (pre-test and post-test) in each group is considered normally distributed ($p > 0.05$). Finally, the results of the study were presented according to the research question and hypothesis formulated.

4.1 Students Achievement in Mathematical Thinking Test Before Intervention of Problem-Solving Approach

Table 2 indicates that the experimental groups had a mean achievement score of 6.84 and a standard deviation of 3.54 in the pre-test. At the same time, the control group had a mean and standard deviation of 6.48 and 2.99, respectively. The finding in Table 3 shows no significant difference in the pre-test scores of the experimental and control groups [$t(48) = 0.39, p > 0.05$]. This finding means that both groups had almost similar achievements in the pre-test before implementing the intervention. In addition, both groups (experimental and control) have low achievement in the pre-test of mathematical thinking test since both groups showed values of less than half from the total score.

Table 2

Descriptive Statistics of Students Overall Achievement in Pre-Test

Test	Type	N	Mean	Std. Deviation
Pre-Test	Experimental	25	6.84	3.54
	Control	25	6.48	2.99

Note: Full mark = 40

Table 3

Independent Sample T-Test (Pre-Test)

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2- tailed)	95% Confidence Interval of the Difference	
Pre- Test	Equal variances assumed	0.196	0.660	0.388	48	0.699	-1.504	2.224
	Equal variances not assumed			0.388	46.67	0.700	-1.505	2.225

4.2 Effects of Problem-Solving Approach on Students' Achievement in Mathematical Thinking Test

Shapiro-Wilk and the Levene's test showed that the assumption of normality was met within each group. Box's Test of Equality of covariance matrices was not significant value ($p > 0.05$), and thus, parametric tests can be applied. Table 4 indicates the result of the pre-test and post-test for both groups (experimental and control). During the pre-test, the experimental group had a slightly similar mean score of 6.84 (s.d=3.544) with the control group with a mean score of 6.48 (s.d=2.99). However, for the post-test, the mean score for the experimental group (mean=23.4; s.d=9.33) was higher than the control group's mean score (mean=8.32; s.d=4.61). Thus, the mean value of the experimental group outperformed the control group during the post-test. The experimental group showed a marginal increase of mean value during the post-test. For the control group, the mean value increased over time; however, the growth of the mean value of experimental groups was much better than the control group.

Table 4

Descriptive Statistics for the Two-Groups on Achievement in Mathematical Thinking Test

Test	Type	Mean	Std. Deviation	N
Pre-Test	Experimental	6.84	3.54	25
	Control	6.48	2.99	25
	Total	6.66	3.25	50
Post-Test	Experimental	23.40	9.33	25
	Control	8.32	4.61	25
	Total	15.86	10.54	50

Note: Full mark score=40.

A split-plot analysis of variance (SPANOVA) was conducted to assess the effect of the intervention and a control group on participants' achievement in mathematical thinking tests across the two time periods of pre-post intervention. Table 5 indicates the value of sphericity assumed shows that there is a significant effect in mathematical thinking test (pre-post tests) [$F(1,48)=82.37, p<0.05$]. Furthermore, there was a significant interaction between groups (experimental and control) and mathematical thinking test (pre-post tests) [$F(1,48)=52.21, p<0.05$]. Thus, the null hypothesis is rejected and reported that PSA intervention affects the participants' achievement in mathematical thinking tests.

Table 5

Tests of Within-Subjects Effects

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
PreTest-PostTest	Sphericity Assumed	2125.210	1	2125.210	82.37	0.00
PreTest-PostTest*Type	Sphericity Assumed	1346.890	1	1346.890	52.21	0.00
Error (PreTest-PostTest)	Sphericity Assumed	1238.400	48	25.800		

The comparison results between the tests (pre and post) and groups (experimental and control) were indicated in Tables 6 and 7. Tests of Within-Subjects Contrasts (Table 6) suggests that, overall, there is a significant correlation between pre-test and post-test (plotted in profile plot in Figure 1). Table 7 indicates that mathematical thinking test mean scores were significantly different between experimental and control groups [$F(1,48)=38.19, p<0.05$].

Table 6

Tests of Within-Subjects Contrasts (Overall Achievement)

Source		test	Type III Sum of Squares	df	Mean Square	F	Sig.
PreTest-PostTest		Linear	2125.210	1	2125.210	82.372	0.000
PreTest-PostTest *		Linear	1346.890	1	1346.890	52.205	0.000
Type							
Error(test)		Linear	1238.400	48	25.800		

Table 7

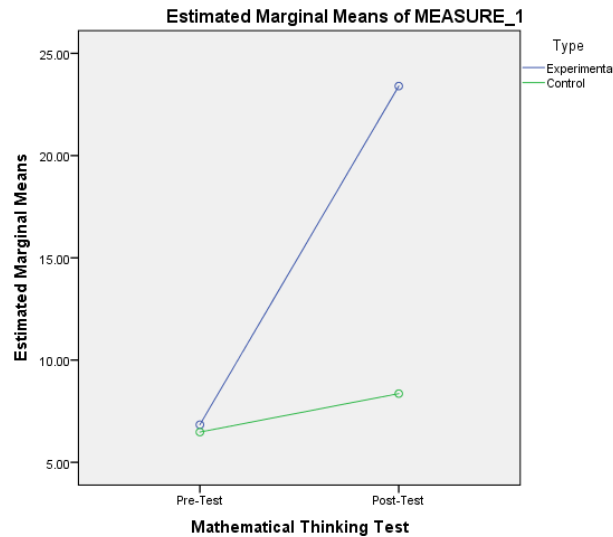
Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	6350.65	1	6350.65	327.25	0.000
Type	741.13	1	741.13	38.19	0.000
Error	931.48	48	19.41		

Figure 2 clearly shows an interaction effect of mathematical thinking scores between the treatment and experimental groups. It was indicated that the mean score of the experimental group (experienced in PSA) is linearly increased before and after the intervention. In contrast, the control group's mean score (without the intervention) has hardly changed their achievements. The graph The results imply that the intervention of the problem-solving approach enhanced participants' achievement in mathematical thinking. This graph (Figure 2) proves that problem-solving approach interventions effectively improve student performance in mathematical thinking.

Figure 2

Profile Plot for Achievement of Mathematical Thinking Test Scores of the Two Groups



5. Discussion and Conclusion

The study aimed to investigate the initial level of first-year undergraduate university students (majoring in mathematics courses) attainment level in mathematical thinking test before PSA intervention. Besides, this study examines the effect of the problem-solving approach on students' development of mathematical thinking. The study's first finding showed that both groups (experimental and control) have no significant difference in their pre-test of mathematical thinking test [$t(48) = 0.39, p > 0.05$]. Both groups obtained low scores, experimental group with 17.1% (mean=6.84, max score=40) and control group 16.2% (mean=6.48, max score=40) respectively in the pre-test of Mathematical Thinking test. Based on the students' mathematics background, they have obtained an A grade in Modern Mathematics. More than 50% in each group had good grades (grade B and above) in Additional Mathematics in the national examinations. However, this good grade achievement in their national exam is not the same in their mathematics thinking test even though its content is similar to high school level.

This finding of low attainment performance can be correlated with previous studies that stated novice undergraduate students lack intellectual capacity when they enter university and college (Singh et al., 2018; Atuahene and Russell, 2016; Singh et al., 2016). The test given to the participants involved consists of non-routine problems. A non-routine problem is any complex problem that requires some degree of creativity or originality to solve. Non-routine problems are problems for "which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example" (Stein and Lane, 1996). These type of problems that cannot be solved with a known method or formula and require analysis, synthesis, and creativity. Non-routine problems typically do not have an immediately apparent strategy for solving them. The students need to think strategically and adapt their knowledge to the problem (Stein and Lane, 1996). Based on these facts, we could consider that most participants have low proficiency in solving non-routine problems. This finding somewhat correlated with previous research that many present students have a low ability as problem solvers. According to Nasir et al. (2021), the difficulties faced by undergraduate students when solving non-routine problems are difficulty comprehending the question, difficulty relating the problem with possible strategy and prior knowledge, and difficulty verifying the solution to the problem. The same pattern of performance was found by Hoon et al. (2018). They have found several characteristics of undergraduate difficulties and struggle in solving problems are recorded. Firstly, the majority of the participants were weak and faced challenges in understanding the questions. Secondly, the participants

did not show further effort to develop a solution when the researcher requested an alternative solution. Thirdly, some participants practised the strategy of trial and error without deep thinking and planning. Fourthly, it was also found that participants directly applied procedural knowledge and computation without any reasoning and thinking. Lastly, some participants are not able to describe their solution reasoning in a logical and meaningful way. They are just able to explain the algorithmic procedure processes applied. Current studies have found that mathematics learning nowadays is focused on rote learning. The students are found too reliant on memorisation of the formula and rote procedure to use (Boaler and Zoido, 2016; Singh et al., 2016) and have neglected the thinking process when solving the problem. Due to that, grades obtained in the national standardise examination currently do not correlate with students' mathematical knowledge on problem-solving (Singh et al., 2018; Singh et al., 2016) and do not reflect that students are prepared for college-level work (Atuahene & Russell, 2016).

The prime objective of this study is to investigate the effects of PSA in enhancing the development of students mathematical thinking. Based on the result, the students who experienced the eight-week PSA intervention showed a significant increase in the Mathematical Thinking Pre-Test-Post-Test score. Through the application of PSA, the students who underwent the intervention process had enhanced the mean score of 6.84 to 23.40 out of 40, the total mark score. Based on the SPANOVA analysis and comparing the mean scores of both groups (experienced PSA and inexperienced PSA), the intervention of PSA has contributed a significant effect to students mathematical thinking tests (pre-post tests) [$F(1,48)=82.37, p<0.05$]. Besides, there is a significance interaction between the groups (experienced PSA and inexperienced PSA) with their mathematical thinking test (pre-post tests) [$F(1,48)=52.21, p<0.05$]. In other words, the intervention of PSA has positively affected the students' achievement in the mathematical thinking test scores.

Polya (1973) suggested that teaching instruction based on the problem-solving method can develop the students' ability in mathematical problem-solving. Problem-solving requires and encourages students to use higher-level cognitive abilities such as creativity, analysis, synthesis and evaluation when simplifying the tasks. As such, Polya's (1973) four stages of problem-solving (understanding the problem, devising a plan, carry out the plan and looking back) and Schoenfeld's (1992) six processes of problem-solving (reading, analyse, explore, plan, implementation and verification) need to be considered. Each of these phases has its importance. According to Rasid et al. (2020), classroom culture plays a crucial role in fostering effective instructional practices in STEM education. To enhance students' mathematical thinking, it is essential to cultivate a learning environment that encourages critical thinking and incorporates elements of mathematical problem-solving, ultimately improving their ability to think mathematically. In this study, students were explicitly taught to apply the problem-solving stages by Polya (1973) and Schoenfeld (1992) together on the various strategies (heuristics) to solve a given problem. A problem-solving process begins with the first encounter with a problem and concludes with discovering an answer based on the available information (Olaniyan & Omosewo, 2015). This process has exposed and familiarised students with cognitive-metacognitive strategies indirectly. Yildirim and Ersözölü (2013) mentioned that problem-solving requires students' involvement in cognitive and metacognitive processes. During the intervention, these students were provided with prompting questions for each stage of problem-solving to apply these concepts of cognitive-metacognitive strategies more effectively and practically. The finding has shown a drastic improvement of achievement in mathematical thinking tests among the intervention students. This finding corroborates with the comprehensive spread agreement about problem-solving roles forming students' metacognitive behaviour towards successfully solving mathematics problems (Hassan & Rahman, 2017; Nasarudin Abdullah et al., 2014; Yildirim and Ersözölü, 2013; Schoenfeld, 2013) pointed out that awareness about the metacognitive can encourage students to apply specific strategies to complete a task given and positively impact their achievement. According to Nasarudin Abdullah et al. (2014), problem-solving is a complex process. Therefore, students are expected to integrate some of cognitive and metacognitive to find the solution. Metacognition is mainly concerned with certain processes of students thinking required to solve problems for which no fully developed or automated procedures are available (Schoenfeld, 2013). Rather than determining the correct answer, problem-solving involves comprehending and controlling more complex metacognitive strategies such as planning, monitoring, and evaluation. Self-regulation (planning, monitoring and evaluation) will help students enhance the skills needed to solve problems (Nasarudin Abdullah et al., 2014). On the other hand, it will avoid the worst that may happen, such as embarking upon computations or construction

without having understood the problem, carrying out the details without having seen the connection (sort of planning) and losing the best effects due to failure to reexamine and reconsider the completed solution (Polya, 1973).

In addition, during the intervention, the students are explicitly taught the various problem-solving heuristics. The ability of apply heuristics to assist students in solving mathematics problem is undeniable from the past to the present time in improving students thinking especially in solving problems (Nasir et al., 2023; Singh et al. 2018; Schoenfeld, 2013; Dreyfus, 2002; Schoenfeld, 1992). Nasir et al. (2023) found that incorporating heuristics as an intervention for undergraduates had a positive impact on their problem-solving skills and mathematical thinking. According to Schoenfeld (2013), “strategic competence” is one of the main components to ensure students are proficient in solving problems. This finding reveals that the flexible application of problem-solving strategies (heuristics) improves students’ thinking when solving mathematics problems. Singh et al. (2018) have found that the disclosure of the flexible application of problem-solving strategies (heuristics) to the students seems to aid students in solving mathematics problems. During the intervention, each student is encouraged to provide more than one solution when solving the problem. Vale and Barbosa (2018) have stressed that expecting and encouraging students to provide multiple strategies and solutions would allow the development of students’ flexibility of thinking and, consequently, contribute to their creativity. Dreyfus (2002) supports this argument, stating that heuristics might generate new ideas and, thus, be crucial to recognising the unknown relationship while solving the problem.

The findings from the quasi-experimental pre-post-test design utilised in this work are acknowledged to have certain limitations due to the lack of random student assignment to the experimental group, and the control group is selected via block sampling. Furthermore, conducting randomised controlled trials in this investigation was neither rational nor ethical. As a result, the study’s validity is jeopardised in various ways. As such, results from this study are not aimed to be generalised to a population. However, the finding could represent ‘realistic’ conclusions in educational settings.

In conclusion, the findings have indicated that both groups (experienced PSA and no experience of PSA) had low-level mathematical thinking attainment at the beginning of the study even though most of them obtained good grades in the national mathematics examination. However, after eight weeks of intervention PSA, there was a noticeable improvement among the students who had experienced PSA compared to those who did not. Thus, there is a high probability that methods used and emphasis in mathematics do not emphasise thinking capacity and solving application problems. Many have testified that the standard approach practised by most teachers and mathematics instructors in the classroom is based on rote learning (Singh et al., 2019; Singh & Hoon, 2017). The “drill-and-skill” strategy is still used in most mathematics classes (Bowyer & Darlington, 2016). The impact could be seen through the trend of students’ performance in the past research (Hoon et al., 2018; Singh et al. 2016; Zakaria et al. 2009) and the initial stage of this study where the students discovered struggles when solving practical and real-life problems, hence, the majority achieved a lower score.

This study shows that the application of problem-solving has a positive effect on the development of novice undergraduate students. Problem-solving has proven to provide students with tools to apply their mathematical knowledge to solve hypothetical and real-world problems (Polya, 1973). The basic principle of the problem-solving approach is to nurture students learning of mathematics by/for themselves. This approach aims to develop students who think and learn mathematics by/for themselves (Isoda & Katagiri, 2012). This will encourage students to believe in their ability to think mathematically. They will realise that they can apply the mathematics they are learning to solve the problem. We believe that introducing the Problem-Solving Approach application as a supplementary course for novice undergraduate students would greatly benefit their cognitive-metacognitive growth in mathematics learning in future. The problem-solving approach used in this study was successful because the learning experiences exposed students to stimulating problem situations, propagated the generation of fundamental mathematical ideas, conscious with their mind and required them to use the learned strategies (heuristics) to solve the assigned problems. By being familiar with the problems which need the usage of critical thinking and logic based on the fundamental conceptions of mathematics, plus with the systematic instruction and guideline of problem-solving, would develop not just mere mathematics skills. This problem-solving application will not compete with the other mathematics courses offered by most universities. Instead, it will be completing it as a holistic package towards students mathematical thinking development.

6. Author contributions

Author 1 wrote the project administration, conceptualization, data curation, formal analysis, writing. Author 2 composed the methodology, formal analysis ; Author 3 structured the literature review, interpretation and validation of results, writing – review and editing. All authors reviewed and approved the final version of the manuscript.

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8. Ethical statement

This study was approved by the UiTM Research Ethics Committee (Ref. number: REC/04/2021(MR216))

9. Data sharing statement

Data supporting the findings and conclusions are available upon request from the corresponding author.

10. References

- Abdullah, A. H., Rahman, S. N., & Hamzah, M. H. (2017). Metacognitive skills of Malaysian students in non-routine mathematical problem solving. *Bolema - Mathematics Education Bulletin*, 31(57), 310–322. <https://doi.org/10.1590/1980-4415v31n57a15>
- Albay, E. M. (2020). Towards a 21st Century Mathematics Classroom: Investigating the Effects of the Problem-Solving Approach Among Tertiary Education Students. *Asia-Pacific Social Science Review*, 20(2), <https://doi.org/10.59588/2350-8329.1303>.
- Atuahene, F., & Russell, T. A. (2016). Mathematics Readiness of First-Year University Students. *Journal of Developmental Education*, 39(3), 12.
- Boaler & Zoido (2016) Why Math Education in the U.S. Doesn't Add Up. Scientific American November 2016. Retrieved from <https://www.scientificamerican.com/article/why-math-education-in-the-u-s-doesn-t-add-up/>
- Bowyer, J., & Darlington, E. (2016). “Applications, applications, applications”: Lecturers’ perceptions of students’ mathematical preparedness for STEMM and Social Science degrees Executive Summary. (July), 1–29.
- Downing, K., Kwong, T., Chan, S.W., Lam, T.F. & Downing, W.K. (2009). Problem-based learning and the development of metacognition, *High Educ*, 57: 609–621
- Dreyfus, T. (2002). Advanced Mathematical Thinking. In *Advanced Mathematical Thinking* (pp. 25–41). Kluwer Academic Publisher.
- Drijvers, P., Kodde-Buitenhuis, H., & Doorman, M. (2019). Assessing mathematical thinking as part of curriculum reform in the Netherlands. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-019-09905-7>
- English, L. D., & Kirshner, D. (2016). Changing agendas in international research in mathematics education. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 3–18). New York: Taylor & Francis.
- Han, K.-H., & Kim, Y.-O. (2016). The Effect of Polya’s Heuristics in Mathematical Problem Solving of Mild Disability Students. *East Asian Mathematical Journal*, 32(2), 253–289. <https://doi.org/10.7858/eamj.2016.020>
- Hasan, A. (2024). Problem-based Math Learning Strategies to Improve Students’ Problem-Solving Skills. *The Journal of Academic Science*, 1(1), 22-26.

- Hassan, N. M., & Rahman, S. (2017). Problem solving skills, metacognitive awareness, and mathematics achievement: A mediation model. *New Educational Review*, 49(3), 201–212.
- Hershkowitz, R., Schwarz, B. B. & Dreyfus, T. (2001). Abstraction in context: epistemic actions. *Journal for Research in Mathematics Education*, 32(2), 195–222.
- Hoon, T. S., Singh, P., Han, C. T., Nasir, N. A. M., Rasid, N. S. M., & Zainal, N. (2020). An Analysis of Knowledge in STEM: Solving Algebraic Problems. *Asian Journal of University Education*. 16(2), 131–140.
- Hoon, T. S., Singh, P., Han, C. T., Nasir, N. A. M., Rasid, N. S. M., & Yusof, M. M. M. (2018). Mathematical Thinking Attainment among University Students. *Journal of Economic & Management Perspectives*, 12(1), 623–629.
- Isoda, M., & Katagiri, S. (2012). Introductory Chapter: Problem Solving Approach to Develop Mathematical Thinking. *Mathematical Thinking*, 1(2012), 1–28
- Mason, J., Burton, L., & Stacey, K. (2010). *Thinking Mathematically* (2nd Editio). Pearson Education.
- Mayer, R. E. & M. C. Wittrock (1996). Problem Solving Transfer, in R. Calfee, R. Berliner (eds.), *Handbook of Educational Psychology*, Macmillan, New York, 47–62.
- Mevarech, Z. R., & Fan, L. (2018). Cognition, metacognition, and mathematics literacy. In Y. J. Dori, Z. R. Mevarech, & D. R. Baker (Eds.), *Cognition, metacognition, and culture in STEM education* (pp.261–278). Springer.
- Nasarudin Abdullah, Lilia Halim & Effandi Zakaria. (2014). VStops: A thinking strategy and visual representation approach in mathematical word problem solving toward enhancing STEM literacy. *EURASIA Journal of Mathematics, Science & Technology Education*, 10(3): 165 – 174.
- Nasir, N. A. M., Singh, P., Narayanan, G., Han, C. T., Rasid, N. S., & Hoon, T. S. (2021). An Analysis of Undergraduate Students Ability in Solving Non-Routine Problems. *Review of International Geographical Education Online*, 11(4), 861–872.
- Nasir, N. A. M., & Singh, P. (2023). The Effect of a Problem-Solving Approach on Students' Heuristics Knowledge Development. *International Journal of Academic Research in Accounting Finance and Management Sciences*, 13(2), 827–842.
- Olaniyan, A.O., Omosewo, E.O. & Nwankwo, L.I. (2015). Effect of Polya problem-solving model on senior secondary school students' performance in current electricity. *European Journal of Science and Mathematics Education*, 3(1), 97 – 104.
- Polya, G. (1973). *How to Solve It* (Second). Princeton, New Jersey: Princeton University Press.
- Rahman, M. M. (2019). 21st Century Skill “Problem Solving”: Defining the Concept. *Asian Journal of Interdisciplinary Research*, (April), 64–74. <https://doi.org/10.34256/ajir1917>
- Rasid, N. S. M., Nasir, N. A. M., Singh, P. & Han, C. T. (2020). STEM Integration: Factors Affecting Effective Instructional Practices in Teaching Mathematics. *Asian Journal of University Education*. 16(1). 56–69.
- Safari, Y., & Meskini, H. (2016). The Effect of Metacognitive Instruction on Problem Solving Skills in Iranian Students of Health Sciences. *Global Journal of Health Science*, 8(1), 150–156. <https://doi.org/10.5539/gjhs.v8n1p150>.
- Schoenfeld, A. H. (1992). Learning To Think Mathematically: Problem Solving, Metacognition, And Sense Making In Mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning*, (pp.334–370). New York: Macmillan.
- Schoenfeld, A. H. (1994). Mathematical Thinking and Problem Solving. In *The Medical world* (Vol. 159). <https://doi.org/10.4324/9781003000013-14>.
- Schoenfeld, A. H. (2013). Reflections on Problem Solving Theory and Practice. *The Mathematics Enthusiast*, 10(1).
- Singh, P., Hoon, T. S., Rasid, N. S., Nasir, N. A. M., Han, C. T., Rahman, N. A. (2016). Teaching and learning of college mathematics and student mathematical thinking : are the lines of the same track? *Asian Journal of University Education*. 12(2). 65–78.
- Singh, P., Han, C. T., Nasir, N. A. M., Ramly, M. A. Bin, & Hoon, T. S. (2016). Factors Contributing to Students' Poor Performance in a Mathematics Preparatory Program. In *7th International Conference on University Learning and Teaching (InCULT 2014) Proceedings*. https://doi.org/10.1007/978-981-287-664-5_28.

- Singh, P., & Hoon, T. S. (2017). Islands of superficial knowledge without a canoe to get from one end to the other: the nature of college mathematics. *International Journal of e-Learning and Higher Education*, 141- 161.
- Singh, P., Teoh, S. H., Cheong, T. H., Md Rasid, N. S., Kor, L. K., & Nasir, N. A. M. (2018). The Use of Problem-Solving Heuristics Approach in Enhancing STEM Students Development of Mathematical Thinking. *International Electronic Journal of Mathematics Education*, 13(3), 289–303. <https://doi.org/10.12973/iejme/3921>
- Singh, P., Abd Moin, M. A. A. B., Veloo, P. K., Han, C. T., & Hoon, T. S. (2019). The relationship between self-regulated learning and mathematics attitude towards college students development of mathematical thinking. *Universal Journal of Educational Research*, 7(10 D), 48–53.
- Singh, P., Nasir, N. A. M. & Teoh, S. H., (2024). The cognitive gap in the mathematical thinking abilities of high school leavers for college: Are they ready? *EURASIA Journal of Mathematics, Science and Technology Education*. 20(11).
- Smith, J. M., & Mancy, R. (2018). Exploring the relationship between metacognitive and collaborative talk during group mathematical problem-solving—what do we mean by collaborative metacognition? *Research in Mathematics Education*, 20(1), 14–36.
- Stanic, G., & Kilpatrick, J. (1988). Historical perspectives on problem solving in the mathematics curriculum. In R. Charles & E. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 1–22). Reston, VA: National Council of Teachers of Mathematics.
- Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2, 50–80.
- Tambunan, H. (2019). The Effectiveness of the Problem-Solving Strategy and the Scientific Approach to Students' Mathematical Capabilities in High Order Thinking Skills. *Journal on Mathematical Education*, 14(2), 293-302.
- Vale, I. & Barbosa, A. (2018). Mathematical problems: the advantages of visual strategies. *Journal of the European Teacher Education Network*, 13, 23–33.
- Viitala, H. (2017). A Case Study on Finnish Pupils' Mathematical Thinking : Problem Solving And View Of Mathematics. *CERME10*. Dublin, Ireland.
- Watters, S., & Logan, P. (2006). “ *I can solve problems .* ” Glasgow City Council, Education Services.
- Wedelin, D., Adawi, T., Jahan, T., & Andersson, S. (2015). Investigating and developing engineering students' mathematical modelling and problem-solving skills, *European Journal of Engineering Education*, 40(5), 557-572.
- Yildirim, S., & Ersözlü, Z. N. (2013). The relationship between students' metacognitive awareness and their solutions to similar types of mathematical problems. *Eurasia Journal of Mathematics, Science and Technology Education*, 9(4), 411–415.
- Zakaria, Effandi. (2003). The effects of cooperative learning on students in a matriculation mathematics class. Unpublished Doctoral Thesis. Bangi: Universiti Kebangsaan Malaysia.
- Zakaria, E., Yazid, Z., & Ahmad, S. (2009). Exploring matriculation students' metacognitive awareness and achievement in a mathematics course. *International Journal of Learning*, 16(2), 333–348.