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Making Sense of Line Graphs

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Abstract: Line graphs serve as visual representations of quantitative information, aiding readers in comprehending extensive data, trends, and relationships. Given their widespread use in daily life, it is expected that everyone can make sense of line graphs. This paper proposes and validates a sense-making perspective, and it reports on the ways mathematics undergraduates make sense of a context-based line graph. Data were collected through a line graph task and semi-structured interviews. The findings indicate that the mathematics undergraduates made sense of the graph through perception, operation, and reason. This paper contributes insights into how the participants make sense of the line graph and offers evidence that can inform researchers, educators and curriculum developers about the validity of the framework. This framework can serve as a useful tool for future research in similar areas and potentially shape instructional approaches.

Keywords: Line graph, Make sense, Operation, Perception, Reason

1. Introduction

Line graphs are widely used as a reductive graphical inscription for encapsulating specific trends over time or explaining the relationship between two or more variables. They serve as powerful tools in various disciplines, including economics, social sciences, natural sciences, and news media (Kwon et al., 2021). Given their widespread use, many national mathematics curricula recognise the significance of developing graph sense among students as this skill offers students an opportunity to learn how to model and solve problems they are likely to encounter in their future careers (Bakri et al., 2021; Berg & Boote, 2017).

However, extracting and processing data and information from graphical formats can be challenging (Peebles & Ali, 2015). According to Glazer (2011), graph interpretation is described as an ability to obtain meaning from graphs. In essence, interpreting a line graph concerns “finding the reference or elaborating its sense that an interpretive community has specified beforehand or can accept as a valid new interpretation” (Roth, 2004, p. 77). Both verbal and spatial cognitive resources contribute to the complexity of line graph interpretations (Fausset et al., 2008).

Existing research has demonstrated that both students and teachers encounter difficulties in making sense of line graphs (Bowen & Roth, 2003; Glazer, 2011; Lowrie & Diezmann, 2007, 2011; Patahuddin & Lowrie, 2019). The literature, however, reveals a scarcity of research on understanding the sense making process of mathematics undergraduates when reading line graphs. Such research is necessary to enrich our understanding of how mathematics undergraduates, who are expected to have more competence and knowledge in processing graphical displays of data, make sense of line graphs.

In light of the importance of making sense of line graphs and the limited research available from the perspective of mathematics undergraduates, the current study focuses on mathematics undergraduates' sense-making of line graphs that commonly reflect time-series data and functional relationships. From a dual aim perspective, this study explores how mathematics undergraduates' make sense of a line graph with respect to cars travel and validates the formulated framework that outlines the different ways of making sense of the line graph. In this article, we want to answer:

1. How do the mathematics undergraduates make sense of the line graph?
2. To what extent does the proposed framework accurately explain the ways in which the mathematics undergraduates make sense of the line graph?

This study used Patahuddin and Lowrie's (2019) Travel Task as an instrument to collect the required data. By addressing the research questions, our study can contribute insights into how the mathematics undergraduates make sense of the line graph and offers evidence that can inform researchers, educators, and curriculum developers about the validity of the framework. This framework can serve as a useful tool for future research in similar areas and can potentially shape instructional approaches.

The theoretical framework for this study is detailed in Section 3. It integrates various existing models and taxonomies to provide a comprehensive understanding of graph interpretation. This framework encompasses three types of sense-making: perception, operation, and reason, as adapted from Chin and Tall (2012). Perception involves recognising objects through sensory inputs, operation focuses on symbolic procedures of calculation and manipulation, and reason involves drawing conclusions based on logical thinking. This framework not only guides the analysis of our data but also provides a basis for evaluating the effectiveness of instructional approaches designed to enhance graph sense.

By incorporating this theoretical framework, our study aims to provide a comprehensive understanding of the sense-making process of mathematics undergraduates when interpreting line graphs. This framework can serve as a useful tool for future research in similar areas and can potentially shape instructional approaches.

2. Prior Research in Making Sense of Line Graphs

In the last three decades, research on line graph interpretation has consistently demonstrated the challenges students face at various grade levels (Friel et al., 2001). Whether at the primary level (e.g., Gattis, 2002; Lowrie et al., 2011, Parmar & Signer, 2005), secondary level (e.g., Kramarski & Mevarech, 2003), or university level (e.g., Ali & Peebles, 2013), students have exhibited erroneous interpretations with respect to line graphs. For example, Parmar and Signer (2005) examined the cognitive processes and understanding about line graphs among 91 fourth and fifth graders, including some students with learning disabilities. They found that the majority of students struggled with understanding axis labels, graph scale, and data relationships. Although it might be common to see students who were weak in constructing graphs had problems in interpreting graphs (Shah & Hoeffner, 2002), Parmar and Signer (2005) evidenced that even students who excelled in graphing also performed poorly in data interpretation.

An example of a study conducted on high school students is the work of Kramarski and Mevarech (2003). They designed a graph interpretation test to examine eighth graders' basic knowledge of linear graphs and their underlying reasoning. Using this test, Kramarski and Mevarech (2003) reported that some students provided correct answers but based them on incorrect reasoning, whereas others explained correctly but arrived at erroneous final answers. These results align with the argument put forth by Postigo and Pozo (2004) that students restrict their interpretation of graphs to reading data and analysing only specific aspects of the graphs.

Issues in interpreting line graphs are not limited to school-level students but are also observed among university students. Ali and Peebles (2013) examined the ability of British undergraduate psychology students to interpret graphs and found that approximately 60% of their students were not able to interpret or misinterpreted the line graphs. In a study involving biology major graduates, Bowen and Roth (2003) reported that their participants' interpretations of a population graph and a plant distribution graph were literal. For example, some participants provided brief explanations for the peaks of the graphs, and others were uncertain about the variables represented on the axes. However, it remains uncertain how mathematics undergraduates make sense of line graphs and whether they face the same problems as those encountered by biology graduates in Bowen and Roth's (2003) study.

The literature review reveals several misconceptions related to interpreting line graphs, highlighting the need for further investigation into how mathematics undergraduates make sense of these graphs. First, there is a common misunderstanding regarding the concepts of slope and height. Gattis (2002) found that German first-graders, with no prior graphing experience, associated steeper slopes with faster speeds and higher graph points with greater quantities. Similarly, McDermott et al. (1987, as cited in Glazer, 2011) observed students mistakenly interchanging the values of slope and height, indicating a failure to grasp their underlying meanings. Additionally, students often perceive line graphs as collections of discrete points rather than continuous data. Leinhardt et al. (1990) noted that students read population growth graphs point by point instead of as intervals. Kerslake (1981) found that students counted plotted points when asked about the graph's content, showing difficulty in understanding the continuity of line graphs.

Furthermore, many students view line graphs as literal pictures. Galesic (2011) and Duijzer et al. (2019) found that students interpreted graphs based on visual resemblance to real-life situations, such as seeing ascending and descending lines as hills. This misconception stems from interpreting graphs solely by their visual features instead of understanding the abstract data they represent. Moreover, teachers also exhibit misconceptions in graph interpretation. Confrey and Shah (2021) found that teachers in New Jersey interpreted graphs in a generic manner, while Patahuddin and Lowrie (2019) documented that Indonesian teachers viewed graphs as iconic representations and misunderstood axis relationships. Given that teachers are expected to have strong interpretation skills, these findings are concerning. These various misconceptions raise intriguing questions about how mathematics undergraduates, who possess more experience and knowledge in graphical data interpretation, understand line graphs. Thus, this study aims to investigate how mathematics undergraduates make sense of line graphs.

3. The Framework

Exploring how mathematics undergraduates interpret line graphs is a complex issue that has been addressed by various theoretical models and taxonomies. Shah and Hoeffner (2002) proposed a three-step approach: recognising and encoding graph features, understanding general relationships, and relating these to the disciplinary context. Similarly, Bertin's (1983) semiotic theory breaks down graph reading into three stages: external identification (recognising labels and scales), internal identification (identifying visual elements like bars and lines), and understanding the data through these elements. Curcio's (1987) theory offers another perspective with three levels of comprehension: reading the data (extracting direct information), reading between the data (identifying relationships within the data), and reading beyond the data (making inferences and predictions). These levels represent increasing complexity in understanding graphs, from basic data extraction to advanced inferential reasoning. Research applying Curcio's framework, such as studies by Arteaga et al. (2015) and Jacobbe and Horton (2010), indicates that while teachers may construct complex graphs, their ability to interpret data at the highest level, involving inferences and predictions, remains limited. These findings highlight the need for improved training in graph interpretation. Given these limitations and the diverse theoretical perspectives, there is a compelling need to formulate a new framework that encapsulates the process of how humans make sense of line graphs. This framework would ideally integrate the strengths of existing models while addressing their gaps, providing a more comprehensive understanding of graph interpretation that can potentially enhance educational practices and training programs.

In the process of formulating the framework of the study, we identified three commonly used keywords relating to making sense of line graphs: interpreting, reasoning and understanding. In fact, the term 'sense-making' also requires a clear definition so that we can see how it relates to the other keywords and operate it precisely in the present study. Interpreting is primarily concerned with the process of obtaining meaning from graphs (Glazer, 2011). Reasoning is described as the process of drawing conclusions (Barmby et al., 2009). Skemp (1971) claims 'to understand something means to assimilate it into an appropriate schema'. Schema is a structure of connected concepts which integrates existing knowledge, The National Council of Teachers of Mathematics (NCTM) (2009) defines sense making as 'developing an understanding of a situation, context, or concept by connecting it with existing knowledge' (p. 4).

Sense making is a broad and holistic process that encompasses and extends beyond interpreting, reasoning and understanding. Interpreting focuses on extracting meaning from specific representations like graphs, while reasoning involves drawing logical conclusions. Understanding, on the other hand, refers to the ability to grasp the meaning of information or concepts based on existing knowledge and experiences. Sense-making integrates understanding, interpreting, and reasoning to create a comprehensive and cohesive understanding of a situation, context, or concept. Thus, while interpreting, reasoning, and understanding are important components, sense-making combines these activities to achieve a deeper and more integrated comprehension.

From the theoretical perspective, we adapted the theoretical framework proposed by Chin and Tall (2012) to suit the topic of this study. This involves broadening the notion of 'reason' of Chin (2013) so that it is not restricted to definition and deduction only. The formulated framework of this study involves three types of sense making: perception, operation, and reason, as described in Table 1.

Table 1

Theoretical Framework

Sense making	Descriptions
Perception	Sense making through humans' sensory inputs, building predominantly on humans' ability to recognise objects in physical and mental worlds. This also involves abstract objects. Take for instance, an individual may look at a line graph and noting general upward trend over the observed periods.
Operation	Sense making in the world of symbolism focusing on symbolic procedures of calculation and manipulation. For example, an individual discovers the meaning of the area under the line graph via multiplying the units of x-axis and y-axis.
Reason	Sense making that focuses on drawing conclusions based on logical thinking. In this case, logical thinking may be based on assumptions, conditions, observations, manipulation of objects, definitions or personal conceptions. Bear in mind that what is logical for one person may not be logical for another, as different individuals construct their logic based on different factors. For instance, an individual may reason that two cars will meet at the intersection point of the graph by assuming both cars start at the same place and use the same route, even though the line graph does not provide such information.

4. Materials and Methods

This study utilised the Travel Task and recruited participants from a public university. Data collection involved administering a questionnaire followed by interviews. Data analysis was conducted according to the framework, ensuring rigorous examination of the collected data.

4.1 Context and Participants

A purposive sampling method was employed in this study. Given the importance of making sense of line graphs and the limited research from the perspective of mathematics undergraduates, we selected participants who were Mathematics majors in their final year of study and had prior experience with line graphs. A total of 41 undergraduates majoring in Mathematics and minoring in Economics at a public university in Malaysia were involved. These students were enrolled in a program with a strong focus on Mathematics, covering subjects such as Linear Algebra, Operational Research, Differential Equations, Numerical Methods, Statistical Analysis, Numerical Calculation, Complex Variables Calculus, Advanced Mathematics, Mathematical Modelling, Decision Science, Real Analysis, Statistical Programming, and Object-Oriented Programming.

The research instrument employed true/false statements due to their simplicity and clarity, allowing students to respond efficiently without confusion. This format enabled broad topic coverage within a limited time. The objective nature of true/false statements ensures consistent and unbiased scoring, enhancing the reliability of the results. Additionally, these statements are effective in identifying specific misconceptions, providing valuable insights for educators to address knowledge gaps. The binary format also simplifies data analysis, making it easier to quantify and interpret the findings.

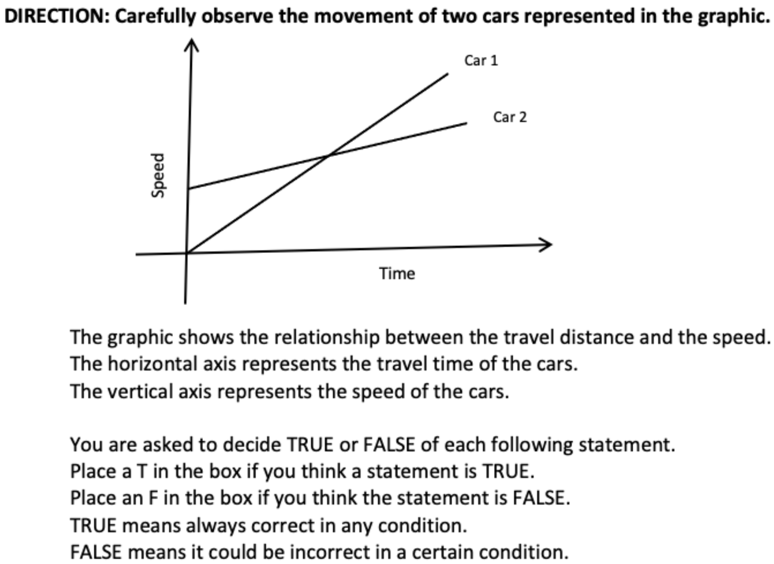
The students participated in the study on a voluntary basis. After receiving approval from the lecturer of the mathematics programme, the researcher was given 10 minutes at the end of a lecture to explain the study. Following the explanation, the questionnaire on the Travel Task (see the next section) was distributed to students who volunteered to participate in the study. Subsequently, an invitation email for a follow-up interview was sent to the students. Building on Patahuddin and Lowrie's (2019) suggestion, future studies on this topic should provide opportunities for participants to explain the reasoning behind their responses. The present study, therefore, implemented follow-up interviews to gain insight into participants' thinking. There were eight mathematics undergraduates participating in the follow-up interviews. In reporting the results, we represent the eight students as S1 to S8, respectively.

4.2 The Travel Task as the Research Instrument

The research instrument is adapted from Patahuddin and Lowrie (2019) so that it could be used to validate the formulated framework and explore how the participants make sense of the line graph. Prior to using this instrument, it was sent to two mathematics education experts for content validation. The research instrument consists of 23 true or false items. Based on Patahuddin and Lowrie (2019), items 1, 2, 7, 13, 16, and 22 were categorised as the most difficult, while items 3, 6, 11, 14, 15, 17, 18, 19, 20, and 21 were categorised as moderately difficult. Items 4, 5, 8, 9, 10, 12, and 23 were categorised as the least difficult. We have incorporated items with varying levels of difficulty to assess the capacity of the formulated framework in elucidating the collected responses across a spectrum of difficulty levels. The task, as illustrated in Figure 1, involves a graph that shows how the speed of two cars changes over time.

Figure 1

The Travel Task (Patahuddin & Lowrie, 2019)



5. Results and Discussion

This section contains the results and discussion of how the mathematics undergraduates make sense of the line graph using the formulated framework. In analysing the collected data, we strive for objectivity by examining not only evidence that aligns with the theoretical framework but also data that contradicts it. Note that this article discusses only 10 items, as they elicited qualitatively distinct responses that encompassed the full range of participants' responses. Furthermore, the data gathered from these items can provide a holistic picture regarding the participants' graph sense-making.

5.1 Most Difficult Items

Of the 23 items, six items (Q1, Q2, Q7, Q13, Q16, Q22) are grouped as the most difficult. Table 2 shows the percentage of mathematics undergraduates with the correct response for these items.

Table 2

Percentage of Mathematics Undergraduates with the Correct Response for the Most Difficult Items

Item	Statement [Answer key: T(true) & F(false)]	Percentage %
Q1	Car 1 and car 2 meet in one place. (F)	48.8
Q2	Car 1 and car 2 meet at the intersection. (F)	41.5
Q7	Car 1 will catch up to car 2 then overtake it. (F)	7.3
Q13	The change in speed of both cars is constant. (T)	39.0
Q16	The distances covered by car 1 and car 2 are different. (F)	31.7
Q22	The graph does not show whether or not car 1 and car 2 meet at a certain point. (T)	48.8

Figure 2

Comparison of Correct Answer Percentages on the Most Difficult Items among the Mathematics Undergraduates

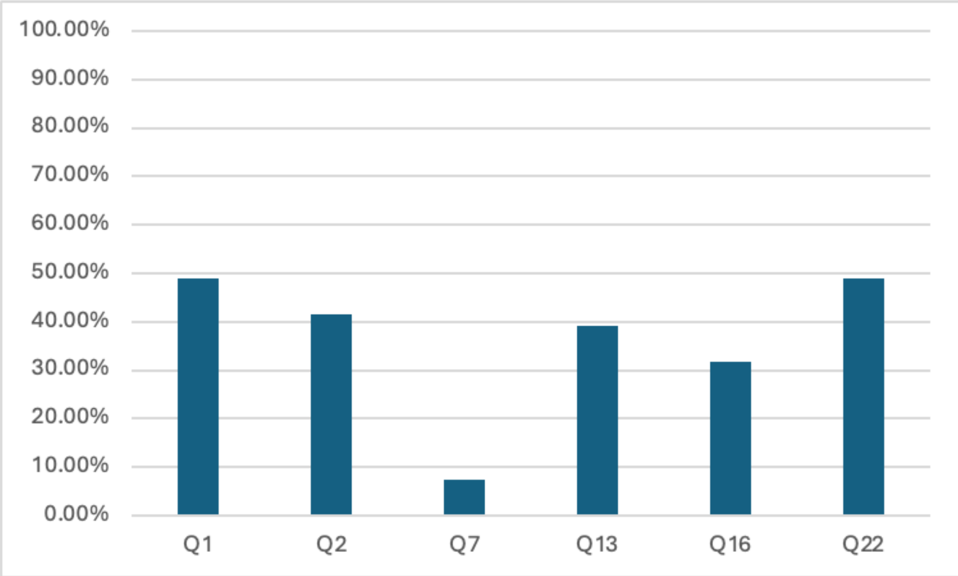


Figure 2 shows the comparison of the percentage of mathematics undergraduates who answered correctly the most difficult items. Significantly fewer participants provided the correct response for Item Q7. Table 2 shows that below 50% of the participants answered each item correctly. Data obtained from Items Q1 and Q22 are not discussed further because they are quite similar to Items Q2 and Q16 that focus on the meeting or intersection point and the travelled distance of the cars. For Item Q2 (car 1 and car 2 meet at intersection), 41.5% of the participants answered correctly. Out of the eight participants who participated in the follow-up interviews, five chose incorrect answers. S7 provided a typical response by saying that, "*As I see from the graph, car 1 is speedier than car 2 so I think they will meet at the intersection after a specific time.*" This indicates that the participant made sense through perception and reason. They saw from the graph that car 1 was speedier than car 2, and then they used logical thinking to conclude that the cars would meet at the intersection point after a specific time. However, this logical conclusion must be based on the condition that both cars are using the same route. Unfortunately, this information is not provided in the graph. The other three participants chose the correct answer and responded that the graph did not show where the cars met. They made sense through perception, looking for information on the graph.

For Item Q7 (car 1 will catch up to car 2 then overtake it), only 7.3% of the participants responded correctly. Out of the eight interviewees, only S6 responded correctly by saying that "*Because I don't know where are the cars, I think they are at different places. But if they are at the same place, car 1 will overtake car 2.*" S6 made sense through reason and was not sure about the locations of the cars as the graph only displayed information relating to speed and time. This shows her awareness of the necessary information or condition to assess the truthfulness of the statement. The other 7 interviewees responded incorrectly by giving slightly different verbal responses as follows:

> *Because after the intersection, car 1 has a higher speed so I think car 1 will overtake car 2. (S3)*
>
> *Because car 1 started at an initial speed of zero and then the acceleration based on the gradient was at a higher gradient compared to car 2 so eventually, I believe that it will catch up to car 2. How I deduce it was based on the formula km/h indicating the speed*

> *of a car so speed times time equal to the distance run by the car. So, when I use this kind of deduction, I believe that the car 1 will overtake car 2, It will go a further distance than car 2. (S8)*

S3 made sense through perception and reason. She saw car 1 had a higher speed. She then made her conclusion based on the assumption that if one car has a higher speed, then it will overtake the other car. S8 made sense through perception and reason. She saw car 1 started at zero speed and had a higher gradient. Then her reasoning was based on comparing the gradients of the two cars. S8 then made sense of the areas under the graphs through operation. She knew that the areas represented the distances travelled by the cars based on the formula of multiplying speed and time. Lastly, she reasoned that car 1 would overtake car 2 due to the point that car 1 travelled a higher distance than car 2. It was highly likely that S8 compared the areas under the car 1 graph and the car 2 graph. It is sensible for us to speculate that S8 must have assumed both cars travel the same route.

For Item Q13 (the change in speed of both cars is constant), only 39% of the participants responded correctly to this item. The analysis shows that three interviewees chose the correct answer and focused on the slope of the graph. S3 provided a typical response and stated that, "*Because they have different gradients, so I think the change in speed will also be different.*" S3 made sense through perception when she saw that both cars had different gradients. She then made sense of it through reason based on this observation, as she realised that the difference in gradients represented a difference in acceleration. The other 4 interviewees provided an incorrect response by giving various reasons.

> *If constant, they will be horizontal but the y axis for both cars is different. (S1)*

> *Because the change in speed for car 1 is constant which means it increased in a same rate but for car 2, it increased slightly. (S5)*

S1 made sense through perception when she noticed that the lines on the line graph were not horizontal. She then made sense of the statement through reason, based on the assumption that if changes in speed for both cars were constant, then the lines on the graph should be horizontal. She was confused about the meaning of the slopes of the lines on the graph. To S5, she must have misunderstood the statement by assuming that Item Q13 was asking if the change in speed of both cars was the same.

For Item Q16 (the distances covered by car 1 and car 2 are different), 31.7% of the participants chose the correct response. Although 3 interviewees chose the correct response, none of them provided a valid reason.

> *Based on the graph, to observe both of the cars, there must be something which is constant. The distances covered by both cars should be the same based on my thinking. Since the speed and time varied, the distance should be constant. (S1)*

> *I think the distances covered by both cars are the same because they have the same destination so they must start at the same initial location, just like a competition. (S2)*

> *From the graph, the lines look almost the same so they should have same distance. (S5)*

The excerpts suggest that S1 made sense through perception when she saw that speed and time were varied on the graph. She then made sense through reason, drawing conclusion based on her personal conception that something must be constant in this situation; therefore, the travelled distance should be constant. S2 made sense through reason, drawing a logical conclusion based on her conceptions that both cars had the same destination, same initial starting location, as if they were in a competition (i.e., having the same route). S5 made sense through perception and treated the lines as if they represented the routes travelled by the cars.

5.2 Moderate Items

Ten items (Q3, Q6, Q11, Q14, Q15, Q17, Q18, Q19, Q20, Q21) are categorised as items with moderate difficulty. Table 3 shows the percentage of mathematics undergraduates with the correct response for these items.

Table 3

Percentage of Mathematics Undergraduates with the Correct Response for the Moderate Items

Item	Statement [Answer key: T(true) & F(false)]	Percentage %
Q3	Car 1 and car 2 will meet if the speed and time of the two cars are the same. (F)	51.2
Q6	The speed of car 1 is always greater than the speed of car 2 at any time. (F)	90.2
Q11	Car 2’s speed is constant. (F)	97.6
Q14	The initial location of car 1 and car 2 always differ. (F)	58.5
Q15	Car 1 left a few minutes after car 2. (F)	70.7
Q17	At all time, car 1 covered a greater distance than car 2. (F)	48.8
Q18	Car 1 and car 2 drove to the right. (F)	78.0
Q19	Car 1 had a steeper route than car 2. (F)	61.0
Q20	The initial position of car 1 and car 2 cannot be identified from the graph. (T)	82.9
Q21	The route travelled by car 1 and car 2 cannot be identified from the graph. (T)	92.7

Figure 3

Comparison of Correct Answer Percentages on the Moderate Items among the Mathematics Undergraduates

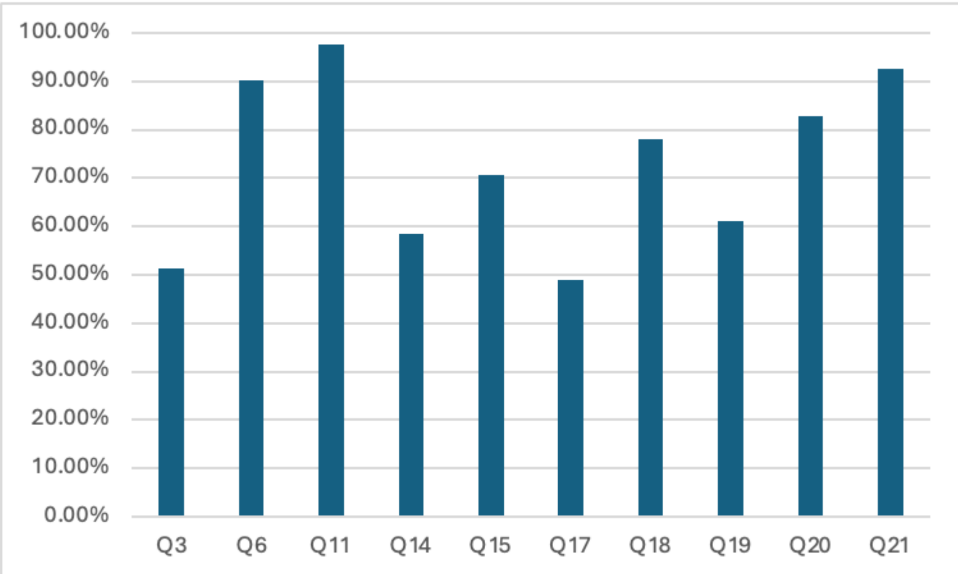


Figure 3 shows the comparison of the percentage of mathematics undergraduates who answered correctly the moderate items. Significantly fewer participants provided the correct response for Items Q3 and Q17. Data collected from Items Q14, Q15, Q17 and Q19 are reported in this section. These items were selected because they involved various aspects of a situation. As an illustration, Items Q14 and Q15 entailed initial location and timing of the cars; Item Q17 involved distance; and Item Q19 is about the travelled route of the cars. For Item Q14 (the initial location of car 1 and car 2 always differ), 58.5% of the participants chose the correct answer. In the interview, five interviewees chose the correct response with four of them providing a similar reason such as "*From the graph, we don't know the initial location of the cars.*" They made sense through perception, noticing the graph did not provide the initial location of the cars. Another interviewee gave an incorrect reason by stating that "*Based on my thinking, because the destination is the same, so the initial location must be the same as well.*" She made sense through reason, purely based on her personal conception. The other three interviewees chose an incorrect answer and gave a similar reason. For example, S4 provided a typical reason by saying that "*Because they started at different points based on the graph.*" She made sense through perception and treated the graph as a map.

For Item Q15 (car 1 left a few minutes after car 2), 70.7% of the participants answered this item correctly. All the interviewees gave a correct answer to this item and provided various reasons as follows:

> *I think both of the cars left at the same time. They started at the same time but with different speeds. (S1)*
>
> *I assumed that the time is the same, they started at the different places but at the same time. (S6)*

S1 made sense through perception by referring to the information presented in the graph. From the graph, she could see that car 1 and car 2 started with different speeds. S6 made sense through reason, drawing a conclusion based on her assumption that the cars started at different places but at the same time.

The analysis for Item Q17 (at all time, car 1 covered a greater distance than car 2) reveals that 48.8% of the participants responded correctly to this item. Out of the eight interviewees, four chose a correct answer and gave a similar reason. For example, S3 and S6 explained that,

> *Before the intersection point, the distance covered by car 1 should be smaller than car 2 because car 1 only catches up car 2 after the intersection of the lines. (S3)*
>
> *Because at the initial point, car 2 has a greater speed so I think car 2 will move faster so car 2 covered more distance. But after the intersection point, car 1 will have a greater distance, so car 1 is not always covered a greater distance. (S6)*

S3 made sense through perception initially, noticing the speeds of car 1 and car 2 before and after the intersection point on the line graph. She then made sense of the statement through reason, drawing a conclusion based on her conception that a car with a higher speed should cover more distance. Similarly, S6 made sense through perception, observing the speed of both cars before and after the intersection point on the graph. She then made sense through reason, drawing a conclusion based on a conception which is similar to S3. S8 chose a correct answer and responded:

> *The item said at all time but I don't think is at all time, it might be at certain time, maybe after the intersection, car 1 might cover a longer distance than car 2 but as I said just now, when the speed times time before the intersection, car 2 is actually higher than car 1, car 2 is actually covered more distance than car 1. (S8)*

S8 made sense through reason when she interpreted the meaning of "at all times" and focused on her conception that car 1 covers a longer distance after the intersection point. This conception was built on her perception and operation. From the graph, she saw that car 2 had a higher speed before the intersection point, and multiplying a higher speed with time will yield a greater distance. This means car 2 covered a greater distance than car 1 before the intersection point. Three interviewees, S2, S4 and S7 chose an incorrect answer for Item Q17. S2 and S7 gave a similar reason by saying "*From the graph, car 1's speed is greater than car 2 so I think that car 1 covered a greater distance than car 2.*" They made sense through perception, noticing from the graph that the speed of car 1 was greater than that of car 2. They then made sense through reason, drawing a conclusion based on the conception that a car with a higher speed will cover a greater distance than a car with a lower speed. The problem was that they did not pay enough attention to the given statement, which stated "at all times". S4 defended her answer by claiming "*Because area under the line of car 1 is larger.*" She made sense through perception when she noticed that the area under the line of car 1 is larger. Again, her mistake was she did not pay enough attention to the provided statement, which stated "at all times".

For Item Q19 (car 1 had a steeper route than car 2), 61.0% of the participants answered the item correctly. Six interviewees gave a correct answer to this item and provided a similar reason. S3 provided a typical reason and stated that, "*Because the graph didn't indicate how steep the route is.*" S3 made sense through perception as they could see from the graph that no information on the travelled route was displayed on the graph. Two interviewees gave an incorrect answer to this item by stating "*Because car 1 has a steeper line than car 2.*" They made sense through perception and treated the graph as a literal picture.

5.3 Least Difficult Items

There are seven items (Q4, Q5, Q8, Q9, Q10, Q12, Q23) with the least difficulty level. Table 4 shows the percentage of mathematics undergraduates with the correct response for these items.

Table 4

Percentage of Mathematics Undergraduates with the Correct Response for the Least Difficult Items

Item	Statement [Answer key: T(true) & F(false)]	Percentage %
Q4	Car 1 and car 2 have different initial speeds. (T)	95.1
Q5	Car 1 had an initial speed of 0 whilst car 2's initial speed was greater than 0. (T)	92.7
Q8	Car 1's speed will be greater than car 2's speed after a certain time. (T)	95.1
Q9	The change in speed for car 1 and car 2 differs. (T)	100
Q10	Car 1's speed continuously increases. (T)	100
Q12	The change in speed of car 1 is greater than car 2. (T)	97.6
Q23	The final destination of car 1 and car 2 cannot be identified from the graph. (T)	85.4

Figure 4

Comparison of Correct Answer Percentages on the Least Difficult Items among the Mathematics Undergraduates

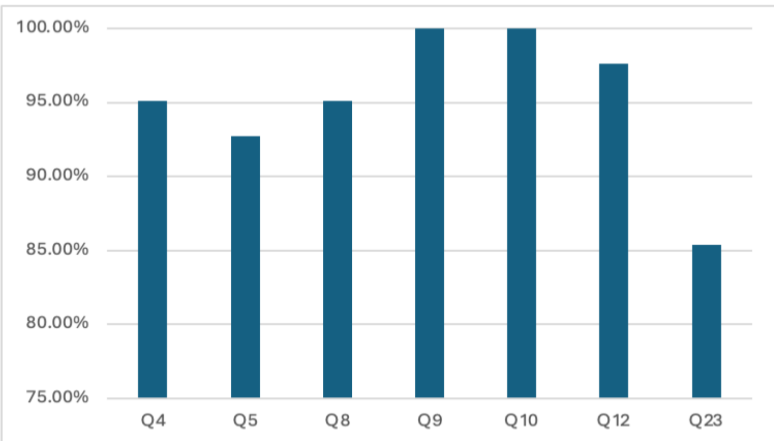


Figure 4 shows the comparison of the percentage of mathematics undergraduates who answered correctly the least difficult items. Significantly fewer participants provided the correct response for Item Q23. All the participants answered Items Q9 and Q10 correctly. Data collected from Items Q4, Q8, and Q23 are reported in this section. These three items were chosen because they represented different aspects of the situation. In this case, Items Q4 and Q8 involved speed and Item Q23 entailed destination. For Item Q4 (car 1 and car 2 have different initial speeds), 95.1% of the participants answered this item correctly. All the eight interviewees gave a correct answer and responded similarly by saying that, "*Looking at the y axis on the graph, car 1 started at zero and car 2 started here, so they have different initial speeds.*" They made sense through perception, noticing the initial speeds of the cars by looking at the y-axis.

For Item Q8 (Car 1's speed will be greater than car 2's speed after a certain time), 95.1% of the participants answered this item correctly. All the interviewees chose a correct answer for this item and gave a similar reason by referring to the graph. S3 provided a typical reason and said, "*after the intersection, car 1 has a higher speed because the line for car 1 is above car 2.*" This shows that the interviewees made sense through perception.

The analysis of Item Q23 (the final destination of car 1 and car 2 cannot be identified from the graph) reveals that 85.4% of the participants responded correctly to this item. Out of the eight interviewees, seven chose the correct answer. Six of the interviewees who chose the correct answer gave a similar reason. S1 provided a typical reason and stated that, "*No information on destination is given in the graph. Only speed and time are provided.*" This shows they made sense through perception, noticing the information provided by the graph. S6 provided an incorrect reason by saying that, "*As initial location is different, the final destination is also different.*" S6 made sense through reason, drawing a conclusion based on her conception that the cars started at different initial locations, then the final destinations should be different. We speculate that she made sense through perception by treating the graph as a map. The cars didn't start at the same point on the graph therefore she perceived that they had different initial locations. One interviewee chose an incorrect answer and responded that, "*The graph can tell the final destination, because we can see the start time and where they stopped.*" She made sense through perception and treated the graph as a map.

5. Conclusion

The present study demonstrates that mathematics undergraduates make sense of context-based line graphs through a combination of perception, operation, and reasoning. Participants employed a mixture of these elements to interpret the provided statements, and the proposed framework effectively explains their sense-making processes. However, it is important to note that some participants responded incorrectly by drawing conclusions based on limited perspectives or focusing on specific aspects of the graph without considering all necessary conditions. For instance, to conclude that two cars meet at the intersection point, it is essential to know if they are traveling the same route but the information that was not provided in the graph. Some participants overlooked this critical aspect and based their conclusions solely on the condition that the cars had the same speed at some point, which ultimately led to incorrect answers. This finding highlights the necessity for students to develop a more comprehensive understanding of the factors influencing graphical data interpretation.

The study also has limitations that can guide future research endeavours. Firstly, the instrument utilised in this research consisted of only one graph without numeric values, which may have constrained the depth of analysis and the richness of the data collected. Future research could address this limitation by incorporating additional graphs with various levels of complexity and including numeric data. By doing so, researchers can gather a more comprehensive dataset and explore how participants make sense of different types of graphs, potentially revealing patterns and discrepancies in their interpretation skills. Secondly, the sample for this study was restricted to undergraduates from a specific program at a single university. This narrow focus may limit the generalisability of the findings. Future studies could involve students from diverse academic backgrounds and institutions, allowing researchers to determine whether similar sense-making processes occur across different educational contexts and among different demographic groups.

The findings from this study carry implications for both educational practice and future research. In educational settings, they highlight the importance of teaching students to consider multiple conditions and perspectives when interpreting graphs, particularly in real-world scenarios that require critical thinking and careful analysis. Educators should emphasise a holistic approach to graph interpretation, encouraging students to explore deeper relationships between variables rather than relying solely on surface-level observations. This could involve integrating more real-life examples and contextual problems into the curriculum to foster students' critical thinking skills and enhance their ability to analyse graphical data effectively.

For researchers, this study provides evidence that the framework of perception, operation, and reasoning can be a valuable tool in analysing how students make sense of line graphs. Future research can further validate and refine this framework by applying it to a wider variety of graphical representations and participant groups. Understanding the specific misconceptions revealed by this study also offers opportunities for developing targeted interventions aimed at improving students' graphical literacy. By addressing these misconceptions directly through instructional materials and teaching strategies, researchers can contribute to enhancing students' overall understanding of mathematical concepts and their applications.

In conclusion, while this study contributes valuable insights into how mathematics undergraduates make sense of line graphs, it also opens avenues for future research and instructional improvement. Addressing the study's limitations will not only strengthen the robustness of the findings but also expand our understanding of graphical interpretation in educational settings. By embracing a broader scope of research and instructional strategies, educators and researchers alike can work towards equipping students with the necessary skills to navigate the complexities of data interpretation in an increasingly data-driven world.

6. Suggestions

Future research could incorporate additional graphs and tasks to gather more data and investigate how participants make sense of different types of graphs. Additionally, studies involving university students from diverse programmes and universities could be conducted to examine if they make sense of line graphs in similar ways.

7. Co-Author Contribution

The authors affirmed that there is no conflict of interest in this article. The first author conceptualised the study, designed the research methodology, and conducted the analysis and interpretation of the results. The second author carried out the fieldwork, prepared the literature review, and performed the statistical analysis. The third author oversaw the writeup of the entire article. All authors provided feedback and contributed to refining the analysis and manuscript.

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