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Leveraging Mathematics to Enhance Critical Thinking in Technical Universities

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Abstract: As the modern labour market increasingly demands professionals capable of complex problem-solving and analytical reasoning, the importance of developing these cognitive abilities during higher education has become paramount. This study investigates the role of mathematics education in fostering critical and analytical thinking skills among students in technical universities. The study was conducted over a five-year period (2015-2020) at the Karaganda Technical University, involving 275 students who were divided into control and experimental groups, with the latter receiving specially designed that students in the experimental group demonstrated a higher improvement in these cognitive abilities compared to their counterparts in the control group. Specifically, the average score for positive indicators of critical and analytical thinking increased by 210% in the experimental group, compared to 191% in the control group. Students in the experimental group achieved an average score of 4.05 (on a 5-point scale) in mathematics, compared to 3.60 in the control group. The study also explored various pedagogical strategies for enhancing interest in mathematics, including problem-based learning, reproductive methods, and heuristic approaches. The findings of this research have significant implications for curriculum design and teaching methodologies in technical universities. The research also highlights the need for a holistic approach to mathematics education that goes beyond mere subject proficiency to foster skills that are increasingly valuable in the modern workplace. Future research could explore the long-term impact of these enhanced cognitive skills on graduates' professional success and investigate the potential for applying similar approaches in other educational contexts.

Keywords: Analytical thinking, Cognitive development, Critical thinking, Mathematical education, Student interest

1. Introduction

Currently, the widespread use of mathematical methods has become a necessary condition for improving the quality of scientific research in various branches of science and technology. In order for a specialist to successfully apply mathematical methods in further professional activity, to model various technological and biochemical, economic processes, first of all, it is necessary to have the necessary knowledge and be able to correctly handle the mathematical apparatus. Mathematical methods are playing an increasingly important and necessary role in the training of specialists, as they allow analysis of the results of theoretical and practical activities with a sufficient degree of reliability. They are used to process observational and experimental data, which are not only subject to measurement errors and random interference but also to the influence of internal variability. The modern labour market needs specialists who are able to think critically and analytically, and these qualities can be developed by means of mathematics. Therefore, in the modern educational process of a technical university, it is necessary to increase the level of mathematical knowledge. After analysing the pedagogical literature to identify methods that contribute to improving the quality of knowledge, the study selected the development of interest.

Thus, there was a contradiction between increasing the requirements for the ability to think critically and analytically for graduates of a technical university in the conditions of the modern labour market, and conditions conducive to the development of these qualities. Insufficient elaboration of this problem in pedagogical science and practice has led to the process of identifying conditions that promote the development of critical and analytical thinking. Thus, the idea was formed: successful mastery of the mathematics course in the system of higher professional education would lead to an increase in the level of critical and analytical thinking of future bachelors.

Pedagogical science has accumulated a huge potential for research in the field of improving the quality of the educational process, and the study will focus in more detail on scientific achievements over the past ten years. Issues of professionally-oriented mathematical training at a technical university are currently being considered by Lvova (2016), Toropova (2018), Scholz and Steiner (2015), Abend (2018), Shyshenko et al. (2024). The methodology of higher vocational education in mathematics is the subject of studies by researchers Zabavskaya (2018), Testov and Perminov (2021). The problems of competencies and educational technologies in higher education are dealt with by Henner (2018), Bordovskaya et al. (2020), and Astashova et al. (2020). The quality of higher education, problems and prospects are considered by Varshavskaya and Kotyrlo (2019), Rubin and Soboleva (2021), Maltsev and Repetskii (2020), Toporkova (2020), Kosova and Khalilova (2019).

Modern pedagogical science considers several ways to improve the quality of mathematical knowledge, here are some of them: gender approach (based on the effect as an emotional process, since men are more susceptible to this feeling than women) (Henschel & Roick, 2017); reflexive learning (Mundia & Metussin, 2019); teacher orientation (collective preparation for classes helps teachers to discuss problems and improve pedagogical skills) (Erbilgin, 2019); cognitive mechanism (the study of mathematics is associated with an interventional approach) (Henschel & Roick, 2017; Mundia & Metussin, 2019; Erbilgin, 2019; Geary et al. 2019; Maher et al., 2018; Dowker, 2019); teaching mathematics using technologies (Clark-Wilson & Hoyles, 2019); personalisation (to create situational interest, thereby arousing individual interest) (Hogheim & Rebera, 2015); predictability gradient (personality, attitude, and well-being are important for achievements in mathematics) (Pipere & Mieriņa, 2017); cognitive work (neurocognitive approach) (De Freitas & Sinclair, 2016); model of productive cognitive activity (development of concepts, proof of theorems, problem solving) (Lupu, 2014).

The purpose of the study is the theoretical substantiate the significance of the level of mathematical knowledge of students in the development of critical and analytical thinking skills. The achievement of the intended goal and the solution of the main problem are connected with the hypothesis: if students show interest and have good academic performance in the process of studying mathematics at a technical university, they develop critical and analytical thinking.

2. Materials and Methods

Research methods: theoretical analysis and generalisation of pedagogical experience; conversations; questionnaires; analysis of special and popular scientific literature; use of mathematical statistics methods in processing the results of experimental research. The methodological basis of the study: the concept of problem-based learning and theoretical developments in the field of professional education by prominent researchers: Lerner and Osmolovskaya (2017), Makhmutov (2016). In order to prove the hypothesis put forward that *if* students show interest and have good academic performance in the process of studying mathematics at a technical university, they develop critical and analytical thinking, the study has developed pedagogical conditions for the development of cognitive interest.

It is necessary to increase attention to teaching methods: the maximum development should be given to methods that stimulate the activity of students, educating them with skills of independent work. It is necessary to revise the methodology of lectures and practical classes to strengthen interdisciplinary links with theoretical and special disciplines in a technical university.

One of the leading developers of the problem of development of interest in the learning process – Shchukina (1988) believes that an interesting lesson can be created by the following conditions: the personality of the teacher; the content of the educational material; teaching methods and techniques. If the first two conditions are not always in the power of the teacher, then the latter is the field for the creative activity of any teacher. The main task of a mathematics teacher at a technical university is not just to teach, but to develop students' thinking utilizing their subject. Try, whenever possible, to integrate knowledge, linking the content of the discipline with future professional activity. In this regard, the study reworked the entire course of mathematics: problem lectures; problem tasks of professionally-oriented content; test tasks used in intermediate and final controls were reworked for the purpose of professional orientation; professionally-oriented textbooks for the presented specialities were developed (in paper and electronic format).

To assess the idea that the development of interest in mathematics would entail an increase in the level of mathematical knowledge of students, which will further lead to the development of critical and analytical thinking in the educational process of the university, a scientific and pedagogical experiment lasting 5 years (2015-2020) was conducted. This experiment was conducted based on the Karaganda Technical University, which took 275 students of eight different technical specialities. The first-year students-respondents, studying the discipline "Mathematics", were divided into control and experimental groups of 138 students each. These groups studied according to the same curriculum, however, special methods and tools were introduced for the students of the experimental group, designed to increase their interest in learning mathematics. Both groups of students had the same initial level of knowledge in mathematics, which made it possible to objectively assess the effectiveness of the implemented methods (Abayeva et al., 2016). To assess the level of critical and analytical thinking, a questionnaire consisting of 15 questions in the form of online testing conducted anonymously was developed. All questions had a single set of answers: "Yes-Sometimes-No". The survey was conducted in three stages of the educational process: initially in the first year before starting the mathematics course, after completing the mathematics course as the second stage, and the third stage before presenting the graduation projects.

The first stage began with the start of the academic years 2015-2016 and 2016-2017. It was aimed at assessing the general level of critical and analytical thinking among students in the 1st year of technical specialities of the Karaganda Technical University. After this check, methods and means of developing interest in learning mathematics were introduced into the educational process in the experimental group, the control group continued their education according to standard regulated methods. The second stage began at the end of the 2015-2016 and 2016-2017 academic years, the next questionnaire was conducted to determine the level of development of critical and analytical thinking in the experimental and control groups. This made it possible to obtain intermediate results regarding the level of success in the "Mathematics" discipline among respondents in comparison with the 1st stage of the study. In May of the 2018-2019 and 2019-2020 academic years, while completing their studies at the university, the students passed the third final stage of the questionnaire to determine the level of development of the final level of critical and analytical thinking and to identify the impact of the applied methods and means of developing interest in the study of mathematics.

It is worth emphasizing once again that the same students took the survey starting from the 1st year of study, the final stage of the survey took place in their 4th year of study. As already mentioned, groups of respondents were taken from different years of study initiation (2015 and 2016, respectively). Thus, all stages of the questionnaire were repeated two years in a row for each course separately, according to their year of entry. These measures were carried out in order to ensure the necessary breadth of the sample and the objectivity of the results of the experiment. This sample is justified by the technical direction of the specialities chosen for testing, the common curriculum of the discipline "mathematics" among the selected specialities of the respondents, as well as the existing favourable conditions for experimenting for a long time and within the same educational environment. These factors became decisive in the approach to sample formation, which made it possible to avoid distortions of indicators and obtain relevant research results. The fact that the level of knowledge in mathematics among the studied groups of students in the 1st year of the university is equal, which is due to the uniformity of the school curriculum, deserves special attention. This aspect was confirmed by the survey results of the first stage of the experiment.

3. Results

To determine the level of development of critical and analytical thinking, before studying the discipline “Mathematics”, students were offered a questionnaire (Table 1). The survey was conducted in the form of online testing, incognito. The answers offered to the students were the same for all the “Yes – Sometimes – No” questions, the survey results are presented in Table 2.

Table 1. The questionnaire

No.	Question
1	Are you able to analyse in advance the information that you will need in the future?
2	In your future professional activity, will you check the data provided?
3	Do you often analyse a situation for yourself?
4	Can you predict the options for the development of any situations?
5	Can you observe?
6	Are you able to pick up details while observing?
7	Can you highlight what is important to you in a situation?
8	Can you make decisions in a situation that requires a swift decision?
9	Do you think you can objectively assess the situation?
10	Are you able to substantiate your opinion?
11	Do you ever make a decision “like the majority”?
12	Are you easily persuaded to change your opinion?
13	If you have heard any information, do you trust what you have seen or heard?
14	Do you argue your point of view in communicating with people?
15	Do you know how to logically build a chain?

Table 2. Quantitative results of the initial level of critical and analytical thinking of students

Number of criterion	Specialities in % (control/experimental)								Avg. result C/E (in %)
	Mech	St	Tm	Bt	Ms	Met	Ot	Log	
1 yes sometimes	32/31	40/39	28/30	31/30	37/39	27/26	30/29	28/29	31.6/31.6
no	48/50	45/45	41/40	46/47	37/36	41/40	45/45	46/45	43.6/43.5

	20/19	15/16	31/30	23/23	26/25	32/34	25/26	26/26	24.8/24.9
2 yes	47/46	49/50	46/45	48/46	50/51	45/44	46/45	47/49	47.3/47.0
sometimes	38/39	38/38	40/38	42/42	42/39	38/39	34/36	33/31	38.1/37.8
no	15/15	13/12	14/17	10/12	8/10	17/17	20/19	20/20	14.6/15.2
3 yes	15/12	19/18	17/17	21/22	23/21	22/20	15/14	23/25	19.4/18.6
sometimes	60/61	62/65	62/63	62/59	61/63	60/61	60/58	60/56	60.9/60.8
no	25/27	19/17	21/20	17/19	16/16	18/19	25/28	17/19	19.7/20.6
4 yes	22/23	25/26	25/26	26/26	26/26	21/22	22/19	22/21	23.6/23.6
sometimes	25/22	23/24	20/19	23/24	21/20	24/22	24/24	24/24	23.0/22.4
no	53/55	52/50	55/55	51/50	53/54	55/56	54/57	54/55	53.4/54.0
5 yes	48/49	51/53	46/47	47/46	50/50	46/46	47/46	50/49	48.1/48.3
sometimes	39/40	38/35	39/38	41/42	41/40	38/37	34/36	33/33	37.9/37.6
no	13/11	11/12	15/15	12/12	9/10	16/17	19/18	17/18	14.0/14.1
6 yes	32/29	34/32	30/30	34/33	34/33	31/32	30/29	31/30	32.0/31.0
sometimes	30/31	33/37	41/40	36/36	33/35	35/33	35/36	32/32	34.4/35.0
no	38/40	33/31	29/30	30/31	33/32	34/35	35/35	37/38	33.6/34.0
7 yes	59/60	58/58	57/55	60/59	61/62	57/56	56/55	58/59	58.2/58.0
sometimes	36/36	38/38	33/36	34/34	34/34	37/39	35/36	32/33	34.9/35.8
no	5/4	4/4	10/9	6/7	5/4	6/5	9/9	10/8	6.9/ 6.3
8 yes	36/35	37/37	34/32	35/36	38/37	33/31	32/30	31/32	34.5/33.8
sometimes	39/38	40/39	38/39	39/39	40/41	40/43	38/41	39/40	39.1/40.0
no	25/27	23/24	28/29	26/25	22/22	27/26	30/29	30/28	26.4/26.2
9 yes	28/29	27/29	27/29	28/30	31/32	28/30	27/26	26/26	27.7/28.9
sometimes	50/51	52/52	53/50	50/50	50/48	52/51	49/50	51/53	50.9/50.6
no	22/20	21/19	20/21	22/20	19/20	20/19	24/24	23/21	21.4/20.5
10 yes	68/66	70/70	66/65	69/70	72/74	65/66	65/65	68/69	67.9/68.1
sometimes	27/28	27/28	27/28	26/25	20/20	28/25	28/30	28/25	26.4/26.1

	5/6	3/2	7/7	5/5	8/6	7/9	7/5	4/6	5.7/5.8
11 yes	32/31	30/28	28/32	25/29	28/29	32/32	33/29	32/32	30.0/30.3
sometimes	32/35	40/41	37/35	41/37	34/36	40/39	39/44	41/42	38.0/38.6
no	36/34	30/31	35/33	34/34	38/35	28/29	28/27	27/26	32.0/31.1
12 yes	24/24	24/26	26/25	27/30	23/21	29/30	31/28	28/29	26.5/26.6
sometimes	52/50	50/51	49/50	48/50	48/54	46/45	45/47	31/28	46.1/46.9
no	24/26	26/23	25/25	25/20	29/25	25/25	24/25	41/43	27.4/25.5
13 yes	46/45	47/46	50/53	52/54	52/55	50/53	52/53	54/52	50.3/51.4
sometimes	34/35	35/35	35/35	33/34	36/30	38/33	36/35	31/32	34.8/33.6
no	20/20	18/19	15/12	15/12	12/15	12/14	12/12	15/16	14.9/15.0
14 yes	33/32	35/34	31/33	36/36	31/30	30/32	32/32	36/33	33.0/32.8
sometimes	36/38	38/35	36/34	35/38	37/37	38/35	37/36	34/37	36.4/36.2
no	31/30	27/31	33/33	29/26	32/33	32/33	31/32	30/30	30.6/31.0
15 yes	48/43	52/50	50/50	47/44	51/51	47/48	43/45	44/44	47.8/46.9
sometimes	45/52	44/44	43/40	45/47	43/41	43/44	51/49	48/49	45.2/45.8
no	7/5	4/6	7/10	8/9	6/8	10/8	5/6	8/7	7.0/7.3
Summary data									
Average %	38.0/37.0	39.9/39.7	37.4/37.9	39.1/39.4	40.5/40.7	37.5/37.9	37.4/36.3	38.5/38.6	38.5/38.4
	39.4/40.4	40.2/40.5	39.6/39.0	40.1/40.3	38.5/38.3	39.9/39.1	39.3/40.2	37.5/37.3	39.3/39.4
	22.6/22.6	19.9/19.8	23.0/23.1	20.8/20.3	21.0/21.0	22.6/23.0	23.3/23.5	24.0/24.1	22.2/22.2
Number of students in groups	20/18	19/21	19/23	19/19	9/7	27/24	9/8	16/17	138/137

C./E. – control/ experimental groups*; specialities: Mech* – "Mechanical Engineering"*; St* – "Standardisation and Certification"*; Tm* – "Technological Machines and Equipment"*; Bt* – "Biotechnology"*; Ms* – "Materials Science and Technology"*; Met* – "Metallurgy"*; Ot* – "Organisation of Transportation, Traffic and Transport Operation"*; Log* – "Logistics (Transport)".

As can be seen from the results of the survey conducted at the first stages of studying mathematics among students of the control and experimental groups, the data do not differ either from the speciality or from the group (control or experimental). The next stage of the study was to introduce the presented innovations for students of experimental groups into the educational process. The results of the implementation showed that at the stage of the final control in mathematics, the students of the experimental groups had a higher average score than the students of the control groups. Thus, it was revealed that having the same initial training in the discipline, the students of the control group had lower results than the students in the experimental group. In the 2013-2014 academic year, the average academic performance score was 3.8, and in the 2014-2015 academic year – 3.7 points.

Starting from 2015-2016, when students began to study in the control and experimental groups, there was a sharp difference, since the result of the control group was 3.6 points, but at the same time the results of the experimental group increased to 4 points, the difference between the groups was 0.4 points. The situation was similar in the 2016-2017 academic year, but only the control group had the same average score – 3.6, and the experimental group – 4.1. Thus, the average score of the initial level of mathematical knowledge in the control group was 2.98, in the experimental group – 3.04 (the difference was 0.06 points, which is 1%), and the final level of mathematical knowledge in the control group was 3.60, in the experimental group – 4.05 (the difference was 0.45 or 9%). The above indicates that the students of the experimental groups have a higher level of mathematical knowledge.

The next stage was to assess how this implementation would affect the level of critical and analytical thinking of future bachelors. For this purpose, a questionnaire survey was also conducted under the same conditions as at the beginning of the experiment, the results are shown in Figure 1.

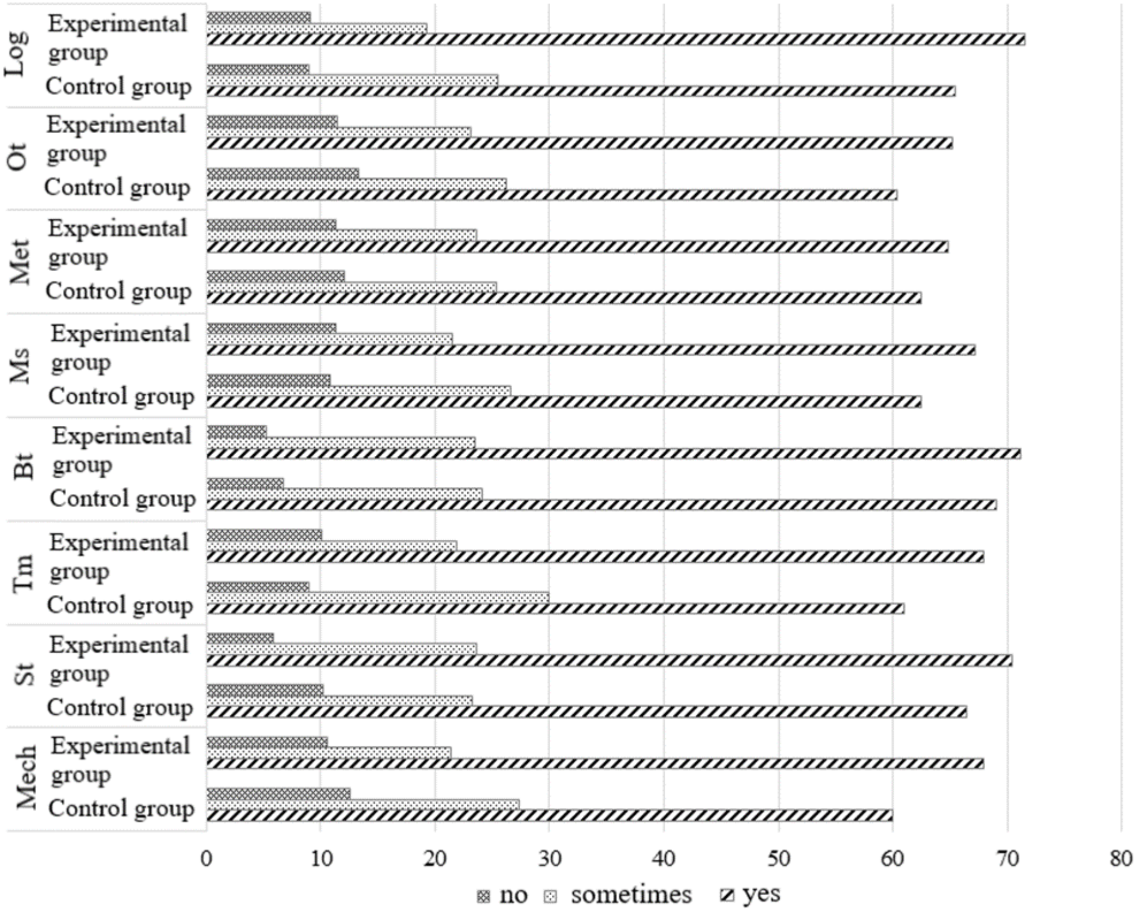

Fig. 1 Results of the study of the intermediate level of critical and analytical thinking of students

The results obtained allow for the conclusion that if students show interest and have good academic performance in the process of studying mathematics at a technical university, they develop critical and analytical thinking, this was the leading hypothesis of this study. Continuing the research in the fact that critical and analytical thinking is developed in the learning process, throughout all 4 years of bachelor's studies, a similar survey was conducted before defending diploma projects from the same students who participated in this experiment in the 1st year. Thus, after 3.5 years (2019, 2020), students passed the questionnaire under the same conditions and questions, the results are shown in Figure 2. Figure 2 shows the results of the initial level of critical and analytical thinking and the final one. Analysing the data, the following conclusion can be drawn: given the same initial level of development of critical and analytical thinking, through the development of interest in mathematics and the introduction of innovative methods of teaching mathematics, sustainable development of the level of critical and analytical thinking has been achieved.

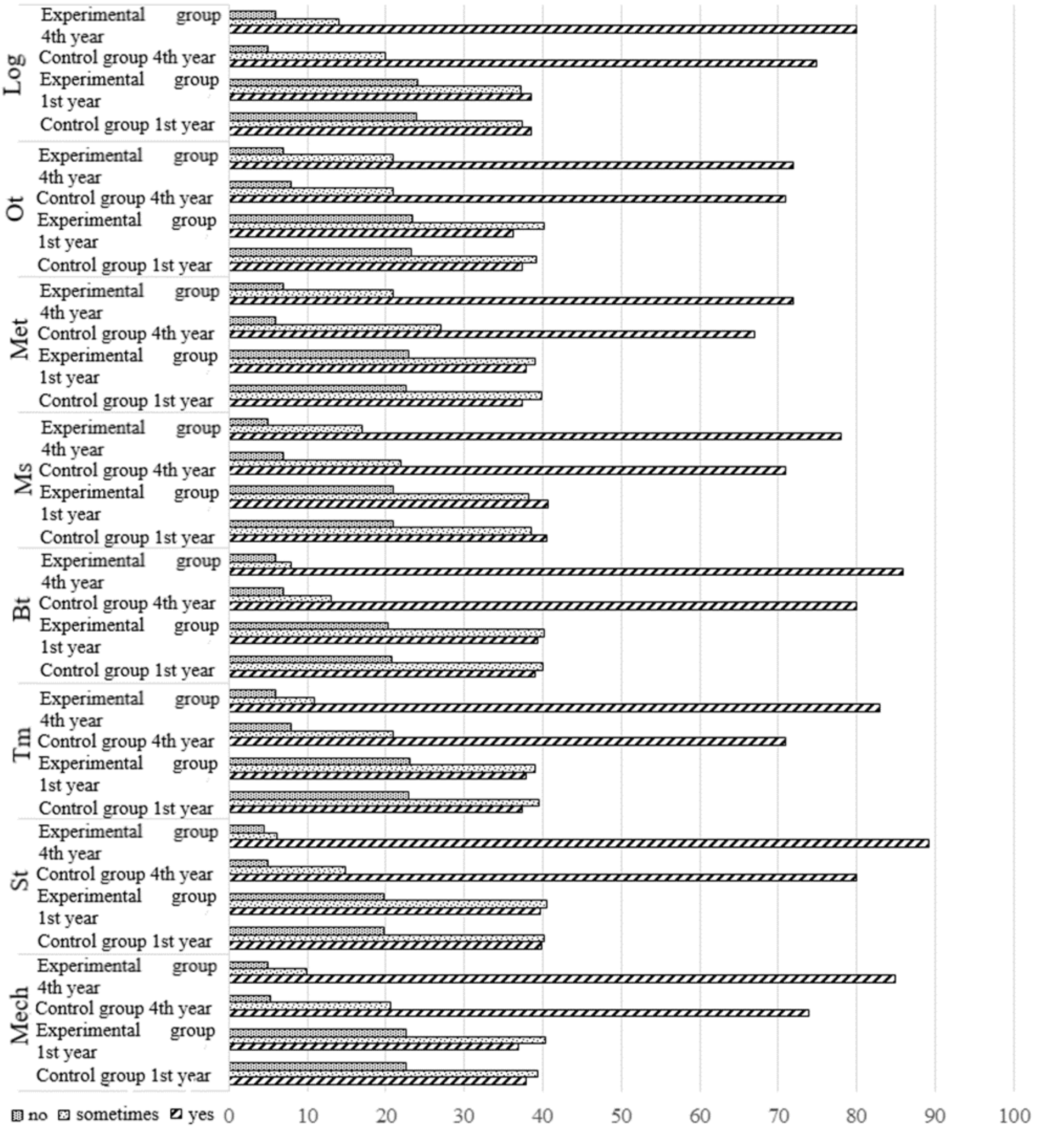

Fig. 2 Results of the study of the final level of critical and analytical thinking of students (in %)

Next, the study will analyse the data obtained in general, without dividing it into specialities.

The "yes" answer: the average score of the experimental group was 38.4% for the answer "yes" in the 1st year, and 80.7% in the 4th year, i.e., the level of critical and analytical thinking increased by 210%; the average percentage in the control group, in the 1st year – 38.5%, in the 4th year – 73.6%, with an increase of 191%. The difference in the results was 29%.

The "sometimes" answer: the result of the experimental group in the 1st year – 39.4%, in the 4th year – 13.5%, the result improved by 34%; the control group in the 1st year chose this answer in

39.3%, and after 3.5 years – 19.9%, the result changed 50.8% for the better. The difference was 16.8%.

The “no” answer: the average score of the experimental group: 22.2% – in the 1st year, and 5.8% – in the 4th year, the results improved by 383%; the average score of the control group in the 1st year was 22.1%, and in the 4th year – 6.4%, the results improved by 345%. The difference was 38%.

Thus, the analysis shows that the level of critical and analytical thinking among students who studied the discipline “Mathematics” developed much better, the average difference was 27.9%. 275 students of Abylkas Saginov Karaganda Technical University (until 2020 – Karaganda State Technical University) of the following specialities took part in the pedagogical experiment: “Mechanical Engineering” (38 students), “Standardisation and Certification” (40 students), “Technological Machines and Equipment” (42 students), “Biotechnology” (38 students), “Materials Science and Technology” (16 students), “Metallurgy” (51 students), “Organisation of Transportation, Traffic, and Transport Operation” (17 students), “Logistics (Transport)” (33 students).

There were 138 students in the control group and 138 students in the experimental group. All students studied according to the same curriculum, but special methods and means of developing interest in the study of mathematics were introduced into the educational process of the experimental group. Thus, having the same initial level of knowledge in mathematics, the experimental data show that the difference between the average values of the progress of the control and experimental groups was 9%, i.e., the students of the control group received the final percentage of progress – 72%, and the students of the experimental group – 81%, if considered on a five-point scale of progress, 3.60 and 4.05, respectively (Figure 3). At this stage of the study, the first part of the hypothesis was confirmed: *if students show interest and have good academic* performance in the process of studying mathematics at a technical university, then they develop critical and analytical thinking, which would be developed in the future and allow them to defend the diploma project more successfully. In order to confirm the second part of the hypothesis about the development of critical and analytical thinking, a questionnaire survey was conducted at three stages of training: at the beginning of the first year, before studying mathematics – stage 1, after completing the study of mathematics – stage 2, stage 3 – before defending diploma projects.

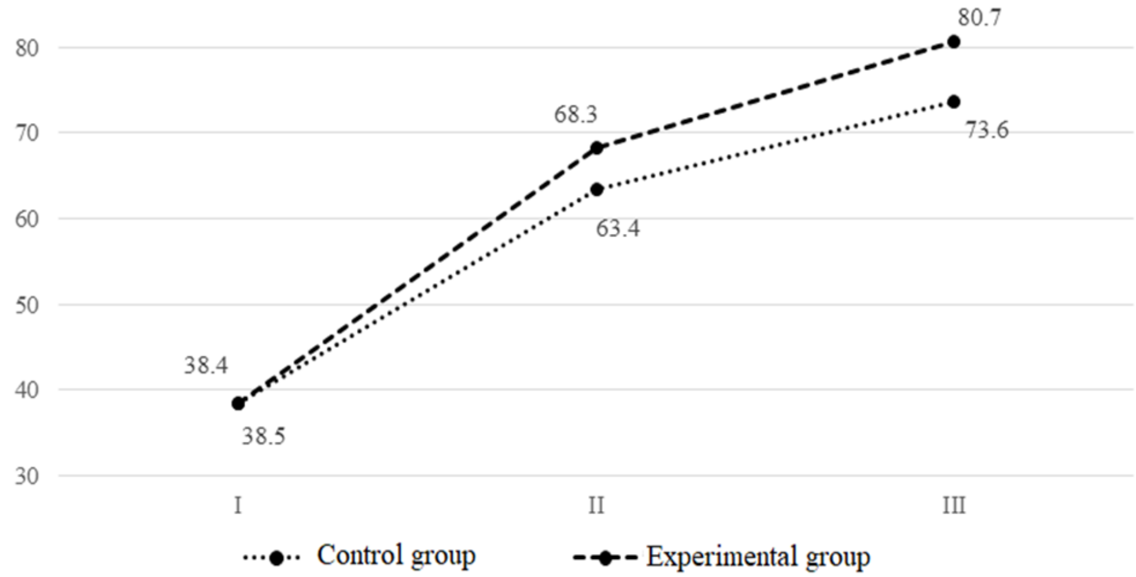

Fig. 3 The level of critical and analytical thinking (in %)

This experiment was started in the 2013-2014 academic year, the first two academic years, the authors investigated the level of academic performance of students of these specialities, studied scientific and pedagogical literature, and determined the purpose, hypothesis, and the main problem of pedagogical research. The next stage was aimed at confirming the hypothesis.

At the beginning of the 2015-2016 and 2016-2017 academic years, the initial level of critical and analytical thinking was tested, and the methods and means of developing interest in the study of mathematics were introduced.

At the end of the 2015-2016 and 2016-2017 academic years, the authors summed up the progress of the control and experimental groups, and the first intermediate results of progress in the discipline “Mathematics” were obtained. At the same time, the second stage of the survey was conducted to determine the level of development of critical and analytical thinking.

In the May 2018-2019 and 2019-2020 academic years, completing their studies at the university, students passed the third stage of the questionnaire to determine the final level of critical and analytical thinking.

Therefore, the scientific and pedagogical experiment lasted 5 years (2015-2020). The goal that was set at the beginning of the experiment: substantiation of the significance of the level of mathematical knowledge of students in the development of critical and analytical thinking skills has been achieved. Hypothesis: if in the process of studying mathematics at a technical university students show interest and have good academic performance, then they develop critical and analytical thinking has been confirmed. Figure 3 shows how the level of critical and analytical thinking has changed in the control and experimental groups at the three stages of the experiment.

4. Discussion

Studying mathematics profoundly influences the development of both analytical and critical thinking, seamlessly weaving these skills into learners' cognitive processes. More than a discipline of numbers and symbols, at its core, mathematics delves into the exploration of patterns, structures, relationships, and problem-solving. Immersing deeply in its tenets not only enhances the capability to dissect intricate issues and recognize underlying patterns, but it also compels individuals to tackle challenges systematically, logically, and with unmatched precision, laying the groundwork for rigorous thought processes. Analytical thinking, epitomised as the capacity to fragment complex issues into their basic elements, is central to mathematical studies. When grappling with mathematical conundrums, students are innately guided to disassemble these problems, pinpointing crucial components, grasping their interrelations, and understanding the foundational principles. Whether it's breaking down an algebraic equation to understand its constituents or delving into calculus to grasp rates of change, mathematics cultivates a mindset that peers beneath the obvious, discerning finer details and nuances (Kamid et al., 2022). Simultaneously, the field nurtures critical thinking – a reflective, evaluative mode of cognition that interrogates assumptions and contests established notions. Every mathematical theorem or proof necessitates solid evidence and a cohesive sequence of statements culminating in a conclusive result (Norezan et al., 2024). Be it validating a geometrical proposition or an algebraic formula; it's not mere acceptance that's sought, but a thorough scrutiny of its elemental truths. This insistence on evidence-driven conclusions propels students to challenge ideas, necessitating justification, authenticating information sources, and affirming that outcomes emanate from cogent reasoning (Ismail et al., 2022).

The multifaceted nature of mathematical problems also exposes students to situations where diverse approaches converge on a solution. Such experiences hone their flexibility in thought, empowering them to assess varied strategies and opt for the most contextually fitting method. This adaptability, coupled with the capacity to switch tactics and reflect on method efficacy, encapsulates the essence of critical thinking (Sari & Juandi, 2023). Moreover, the demands of mathematical precision ensure learners develop a keen discernment. In a domain where ambiguities can skew understanding, precision is paramount. Navigating the labyrinth of numbers and equations reinforces that every minutia counts. This meticulous attention, though rooted in mathematics, translates into a broader life skill, enabling individuals to scrutinize arguments or propositions for their truth, coherence, and validity meticulously (Nasrulloh & Amin, 2022). Mathematics acts as a catalyst for sharpening both analytical and critical skills. The cognitive competencies refined through mathematical exploration not only aid in understanding the universe of numbers but also equip individuals with robust tools to navigate the complexities of the world beyond mathematics.

The challenging nature of mathematics can sometimes be daunting. Yet, when educators harness strategies that sustain student interest, these challenges mould students to persevere and exhibit resilience. Such tenacity is not just limited to mathematical problems but permeates other academic subjects and life's hurdles, instilling in students the invaluable lesson of resilience in the face of adversity. In this regard students, armed with sharpened analytical and critical thinking skills owing to their affinity for mathematics, tend to perform exceptionally well in diverse academic fields. Their ability to critically dissect information, predict logical consequences, and systematically tackle problems is a boon in disciplines ranging from the sciences to the humanities. The infusion of modern technology into mathematical pedagogy, through apps, games, and interactive platforms, not only amplifies student interest but also acquaints them with tools integral to contemporary higher education and professional environments. This technological fluency frequently offers them a competitive edge in both academic and professional arenas (Arisoy & Aybek).

The contemporary approach to teaching mathematics emphasizes the value of collaborative learning. Engaging in group problem-solving sessions provides students with an opportunity to assimilate multiple perspectives, nurturing both their analytical and synthetic thinking capabilities. In an increasingly interconnected global society, the collaborative aspects of modern mathematical pedagogy are vital. When students work together, combining their unique insights and strategies to solve problems, they are not only learning mathematics but also the skills of communication, empathy, and teamwork. Such skills are paramount in fostering a society that values diversity, mutual respect, and shared progress (Patmawati et al., 2019; Didur, 2023). Thus, the strategies employed to kindle a passion for studying mathematics are instrumental in refining students' analytical and critical thinking prowess. Investing in making mathematics more accessible and captivating equips students with cognitive instruments that not only enhance their academic trajectory but also prepare them for real-world challenges, sculpting minds that are rational, discerning, and perpetually inquisitive.

Historically, the essence of mathematics has been intertwined with philosophy, helping seekers answer fundamental questions about existence, patterns, and the universe. Thus, when we foster an enthusiasm for mathematics in students, we're also introducing them to a world of deeper contemplation and inquiry. Such a mindset, which balances the analytical with the philosophical, broadens horizons and encourages students to question, analyze, and deduce in multiple aspects of life, not just within the confines of a classroom. The emotional and psychological benefits of cultivating a genuine interest in mathematics cannot be overlooked (Kravchenko & Srybna, 2023). Facing challenging mathematical problems and conquering them instils confidence, boosts self-esteem, and fosters a growth mindset. It teaches students the art of persistence and the joy of achievement. These qualities, once ingrained, translate to successes beyond academic realms, influencing personal relationships, career paths, and life decisions. In essence, the role of mathematics in shaping young minds extends far beyond equations and algorithms. It's a tool, a philosophy, and a way of life that, when taught with passion and curiosity, has the power to transform individuals and societies alike, steering them toward a future of informed decisions, innovative solutions, and holistic growth (Narot et al., 2014).

The main task of the university is to prepare students to adapt quickly to changing life situations. Students independently acquire the necessary knowledge, at the sight of a problem that has arisen in real life, they learn to find ways to solve it, while developing their critical thinking. Students are able to work correctly with information, use modern technologies, analyse the situation, look for methods of solving problems, and much more. It is necessary to be a sufficiently sociable person who knows how to work in various fields, to contact different social groups, while independently working on the development of thinking. These targets can be achieved only when there is a sufficient amount of knowledge about the nature of critical thinking. The process of teaching mathematics is the development of this pedagogical technology. The social situation that has developed requires a pedagogical rethinking of the role and mechanisms of critical thinking of students. Maltsev and Repetskii (2020) revealed that this aspect is more clearly expressed in the in-depth study of mathematics. The concept of critical thinking can be described in other words as reflexive thinking, and open thinking. Its characteristic is the ability to make deliberate decisions independently, develop arguments, substantiate and take own position, and also an important factor is the ability to listen to the interlocutor, and objectively evaluate own and others' thoughts.

The manifestation of curiosity and the demonstration of personal opinion on a certain issue denote the presence of critical thinking. The student is obliged to defend the results obtained based on their logical arguments and using research methods. The structure of the lesson on critical thinking is quite peculiar and arouses students' interest. After analysing the ways of developing critical thinking, it was revealed that the most effective way is to use mathematical knowledge. The use of mathematical problems increases the level of critical thinking among students. Astashova et al. (2020) proved that the purpose of solving mathematical problems is to teach the student to listen to the opinions of others, and to analyse the logic of their own statements. At the same time, the teacher needs to motivate students for further work and create a creative atmosphere. In the course of solving the problem, students take the initiative, and offer various ideas for solving problems, using forgotten methods and new ways. A partnership is established between students and teachers. Critical thinking develops not only among math lovers but also in students who have a humanitarian mindset.

Modern society is an information society where the flow of information increases several times every day. Since most of the information is unreliable, a person loses the ability to analyse and rethink it. The current pace of development of technical means requires extraordinary human thinking. A person who lacks analytical thinking is likely to have difficulties in life, because of the need to be able to quickly analyse and process large amounts of information. It is only thanks to the analytical and synthetic activity of the brain that it is possible to cut through new information, for this it is necessary to develop analytical thinking. Purposeful development of analytical thinking requires a special organisation of the educational process in educational institutions (Dashko & Miroshnichenko, 2023). It is necessary to create exactly those conditions in which the student will be able to independently solve the tasks set, not be afraid to express a personal opinion in front of classmates, will feel the support of the team, it is necessary to create an atmosphere of goodwill, to make it clear to the student that their words and actions are taken seriously (Teoh, 2023). It is worth mentioning the lectures on mathematics as a way to develop analytical thinking among students. Mathematics is called the gymnastics of the mind. Mathematics plays a huge role in the development of students' thinking abilities. Rubin and Soboleva (2021) found that in the process of solving mathematical problems, students develop the habit of focusing, thinking independently, striving for knowledge, and developing attention. Carried away by the solution to the problem, they do not notice that they are learning and developing their imagination. To solve the tasks set, it is necessary to use ingenuity, guesswork, resourcefulness, and the ability to reason. With the help of systematic involvement in mathematics lectures, students gradually begin to develop analytical thinking.

Geary et al. (2019) mentioned that it is worth considering effective methods that help to develop students' interest in studying mathematics. The first method is a problem-based learning method. This method is characterised by the fact that the learning process proceeds in the form of removing problem situations consistently created for educational purposes. A problematic task means a conscious difficulty that is generated by the discrepancy between the knowledge that is already available and those that are necessary to solve the task. The characteristics of a task are the possibility of an ambiguous solution, the generation of a problem situation, and the willingness and interest of the student to find its solution. There are three types of scientific problems. The first type is the problem of mathematisation, the translation into the language of mathematics of situations and problems, mathematical descriptions that arise outside or inside mathematics. The second type is the problem of studying different classes of models, the purpose of solving this problem is to develop a system of theoretical knowledge using the way of including small theories in it. The third type is the problem of applying theoretical knowledge in new situations and transferring mathematical knowledge to the study of new objects. The basis of problem-based learning is that the teacher does not inform students of ready-made knowledge, but organises students to search for it. Patterns, mathematical concepts, and theories are presented in the course of search, analysis, and observation. The implementation of problem-based learning is successful when there is freedom of choice between the teacher and the student to express their thoughts when the dialogue is carried out in a friendly atmosphere. The second method of increasing students' interest in studying mathematics is the reproductive method. This method consists in the independent search activity of students. The main functions are the education of cognitive interest, the creation of positive motivation for education, the development of the intellectual sphere of personality, cognitive activity, and independence.

Thus, using the research method, students solve the task independently, and the teacher's function is to manage this activity. The research method takes the form of different types of student activities. For example, observation, experiment, modelling, but the essence of this method is the same, that is, independent search activity of students. Using this method, the problem is solved by the students themselves, and the teacher acts as the organiser of the search activity. Next, it is worth mentioning the reproductive method. This method includes the application of the studied material based on a sample or rule. The student's activity is carried out according to instructions, prescriptions, and rules in similar situations, similar to the sample, that is, it has an algorithmic character. The following method is partially a search method, in other words, a heuristic method. It is based on the organisation of an active search for solutions to cognitive tasks put forward in teaching. This can happen either under the guidance of a teacher or based on heuristic programmes and instructions. The process of thinking has a productive character, which is gradually controlled by the teacher or the student based on the work on programmes and textbooks. This method is a proven way to activate thinking, arouse interest in cognition at seminars and colloquiums (Geary, 2019; Kuchai et al., 2023).

Due to the use of the considered methods, it is possible to obtain a high-quality implementation of mathematical education. For the effective conduct of the educational process, an important component is the possession of analytical and critical thinking. Testov and Perminov (2021) stated that it is equally important to mention that the effectiveness of this indicator will depend not only on its possession not only by students but also by the teacher. In order to increase the level of critical and analytical thinking at lectures, it is necessary to organise the learning process by adhering to certain rules. Namely, it is important to revise the formulation of learning goals and objectives in all preparation for classes, update the material in accordance with the set goals and objectives of training, use task complexes to determine the level of critical and analytical thinking skills of students, and use of problem tasks, the purpose of which will be the mathematical preparation of students and the development of critical and analytical thinking.

5. Conclusions

Based on the developed theoretical provisions and the results of experimental pedagogical work, the following conclusions can be briefly drawn.

The study of the level of mathematical knowledge, critical and analytical thinking, revealed the need for its development, since the modern labour market needs specialists who can think critically and analytically, and these qualities can be developed by means of mathematics. The paper analyses the content of mathematical knowledge of students of a technical university in accordance with the future professional activity of students.

The analysis of psychological and pedagogical literature revealed that by developing an interest in mathematics, it is possible to achieve steady growth in critical and analytical thinking. The pedagogical conditions contributing to the development of interest in the study of mathematics are revealed: teaching methods (problematic, reproductive, research, partially-search). The criteria for the development of critical and analytical thinking are defined. As a result of the experimental verification of the effectiveness of the development of interest in mathematics, positive results were obtained, and the level of critical and analytical thinking increased.

Based on the results obtained during the study, the recommendations were developed. The success of the development of critical and analytical thinking depends on the level of mathematical abilities of the student and on the interest in learning; it is advisable to implement methods of developing interest in learning in a complex. Developed pedagogical conditions for the development of interest in the study of mathematics for students of the presented technical specialities can be applied in the educational process of other universities. Criteria for determining the level of critical and analytical thinking can be used in the educational process of any university that prepares technical specialists as a tool necessary for the development of a holistic framework for a specialist.

This study is one of the possible ways to solve the problem of increasing the level of mathematical knowledge and developing critical and analytical thinking in students of a technical university. Further prospects of the study are the following: to check how the level of mathematical knowledge affects the development of an integral structure of a technical specialist; the influence of the level of mathematical knowledge and their development on the quality of students' research projects.

6. Co-Author Contribution

The authors affirmed that there is no conflict of interest in this article. All the authors contributed to the conceptualization of the paper. Author1 overlooked the write-up of the whole article and prepared it for submission. Author2 and Author3 wrote the research methodology and conducted the survey. Author4 and Author5 analysed the data and interpreted it.

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